The Educational Competition Optimizer

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Abstract: In recent research, metaheuristic strategies stand out as powerful tools for complex optimization, capturing widespread attention. This study proposes the Educational Competition Optimizer (ECO), an algorithm created for diverse optimization tasks. ECO draws inspiration from the competitive dynamics observed in real-world educational resource allocation scenarios, harnessing this principle to refine its search process. To further boost its efficiency, the algorithm divides the iterative process into three distinct phases: elementary, middle, and high school. Through this stepwise approach, ECO gradually narrows down the pool of potential solutions, mirroring the gradual competition witnessed within educational systems. This strategic approach ensures a smooth and resourceful transition between ECO's exploration and exploitation phases. The results indicate that ECO attains its peak optimization performance when configured with a population size of 40. Notably, the algorithm's optimization efficacy does not exhibit a strictly linear correlation with population size. To comprehensively evaluate ECO's effectiveness and convergence characteristics, we conducted a rigorous comparative analysis, comparing ECO against nine state-of-the-art metaheuristic algorithms. This evaluation spanned 23 classical functions, 10 CEC2021 test functions, and various real-world engineering design challenges. Empirical findings consistently demonstrate that ECO consistently generates near-optimal solutions across the majority of scenarios, surpassing the performance of the nine other state-of-the-art optimization algorithms tested. ECO's remarkable success in efficiently addressing complex optimization problems underscores its potential applicability across diverse real-world domains. The additional resources and open-source code for the proposed ECO can be accessed at https://github.com/junbolian/ECO and https://aliasgharheidari.com/ECO.html.

Keywords: Global optimization; Metaheuristic; Swarm optimization; Engineering optimization; Genetic algorithm; Educational competition optimizer

1 Introduction

Many industries and sectors encounter a diverse range of optimization problems. These issues frequently involve complications like non-linear characteristics, discontinuities, uncertainties, large-scale dimensions, multiple objectives, and non-convex shapes [1]. This underscores the imperative for advancing more dependable optimization methodologies, mainly focusing on metaheuristic optimization algorithms [2, 3]. These methodologies exhibit stochastic characteristics and can approximate optimal solutions across diverse optimization problems [4, 5]. Significantly, the superiority of metaheuristic optimization algorithms compared to traditional ones is credited to their lack of reliance on gradient information and proficiency in bypassing local optima [6, 7].

Optimization scenarios may encompass numerous objective functions, addressing multiple criteria simultaneously or a single objective aimed at maximizing or minimizing a specific performance indicator [8]. In multi-objective optimization, the exploration of Pareto optimal solutions is crucial for effectively balancing competing objectives [9, 10]. Conversely, single-objective optimization concentrates on swiftly identifying the global optimum within the solution space, typically employing algorithms such as gradient-based techniques or metaheuristic approaches to attain optimal results [11, 12]. Single objective metaheuristic algorithms generally employ two significant search strategies: (i) exploration/diversification and (ii) exploitation/reinforcement. Exploration refers to the ability to explore the search space globally, avoiding local optimality and overcoming local optima traps [13]. Conversely, exploitation involves exploring nearby promising solutions to improve their local quality [14]. Achieving superior performance with an algorithm requires a delicate balance between these two strategies [15, 16]. Compared to traditional methods [17], a fundamental characteristic of population-based algorithms is that they employ simple search and exploitation strategies. However, optimization performance can significantly differ among algorithms that employ distinct operators and mechanisms when confronted with diverse problem scenarios [18].

A widely accepted classification of metaheuristic algorithms delineates them into four distinct classes: evolutionary algorithms, swarm intelligence algorithms, physics-inspired methodologies, and human-derived approaches [19]. Evolutionary algorithms imitate natural evolutionary processes and adopt operators inspired by biological behavior, such as crossover and mutation. A well-known example of this class of algorithms is the genetic algorithms (GA), which draws inspiration from Darwinian evolutionary principles. Other traditional approaches within this category include evolutionary programming [20], ant colony optimizer (ACO) [21], liver cancer algorithm [22], differential evolution [23], and evolutionary strategies [24].

Swarm intelligence algorithms represent an additional category of metaheuristic algorithms, which simulate the collective behavior observed in animal herds or hunting packs [25]. The defining characteristic of these algorithms lies in exchanging information among all group members throughout the optimization process [26]. Notable methods within this category include the RIME algorithm [27], colony predation algorithm [28], Harris Hawks optimization [29], slime mould algorithm [30, 31], whale optimization algorithm [32], weighted mean of vectors [33], parrot optimizer [34], and hunger games search [35].

Physics-based methods form a distinct category of optimization algorithms inspired by the principles of real-world physical laws. These algorithms model the interaction of search solutions through control rules that are anchored in physical processes. Among the notable algorithms in this category are the gravitational search algorithm [36], multi-verse optimizer [37], and charged system search [38].

The final category of optimization techniques includes human-inspired methods, drawing from principles of human cooperation and collective behavior. One frequently employed algorithm in this group is the social cognitive optimization [39], imperialist competition algorithm [40], motivated by human sociopolitical growth practices. Another algorithm within this group is the human evolutionary optimization algorithm [41].

While each algorithm contributes importantly to metaheuristic optimization, they also present specific limitations, which can be summarized as follows:

- **Balancing Exploration and Exploitation**: Metaheuristic algorithms typically involve two main phases: exploration and exploitation. The exploration phase is marked by high randomness, efficient updates, and variable solution quality across iterations. On the other hand, the exploitation phase features algorithmic stability, slower update rates, and consistent solution quality. Achieving the optimal balance between these phases is crucial for maximizing the algorithm's overall performance [42].
- **Parameter Sensitivity**: Parameters play a crucial role in the optimization effectiveness of many algorithms, and identifying the ideal parameters for a specific optimization challenge can be difficult. The absence of qualitative analysis and consideration of parameter sensitivity in newly introduced algorithms makes the task of effectively addressing

complex problems more challenging.

- **Focus on Novelty vs. Computational Performance**: Certain algorithms prioritize novelty by introducing new metaphors without sufficiently emphasizing the computational performance advantages for effectively solving complex problems [27]. This approach can lead to inefficiencies. Additionally, when these algorithms are only tested on a narrow range of problems or a limited set of test cases, they may exhibit high algorithmic complexity and low compatibility. Consequently, these algorithms may not perform effectively when applied to other types of problems, yielding suboptimal results.

Researchers typically do not rely on a single algorithm due to the No Free Lunch theorem [43], which asserts that no single algorithm can effectively address all optimization problems. Hence, it becomes imperative to consider adopting or proposing adjustments to existing algorithms, or even introducing novel approaches, to better tackle present scenarios or provide solutions to evolving challenges. This motivation underpins our proposal of an effective optimization method, the educational competition optimizer.

ECO is an innovative metaheuristic algorithm that draws inspiration from competitive dynamics observed in real-world scenarios of educational resource allocation. It leverages this principle to enhance its search process. The algorithm comprises three phases: elementary, middle, and high school. As an effective human-based optimization model, ECO utilizes an innovative roulette-like structure that iteratively cycles through three distinct phases. This step-by-step approach progressively narrows the range of potential solutions, mirroring the gradual competition within the education system. While ensuring population quality, ECO effectively enhances population diversity and strives to avoid local optima.

In our experiments, we conducted parameter sensitivity analyses and qualitative experiments to elucidate the characteristics and adaptability of the ECO algorithm. We discussed the algorithm's performance under various parameters and when applied to different problem domains. Additionally, to assess the algorithm comprehensively, we compared and tested it against nine highly cited primitive metaheuristics using the 23 classical benchmark functions [44] and 10 CEC2021 test functions [45] test datasets. Furthermore, we verified the algorithm's capacity to solve real-world problems by applying it to five classical engineering optimization problems.

In summary, this paper contributes in the following ways:

1. This research introduces the Educational Competition Optimizer (ECO), an education-inspired meta-heuristic algorithm.
2. This work constructs new exploration, exploitation, and selection mechanisms within ECO, which can be applied to enhance other peer-to-peer algorithms.
3. This paper provides detailed insights into the characteristics of the ECO algorithm through parameter sensitivity experiments and qualitative analysis, facilitating its application to various optimization problems.
4. This paper validates the algorithm's performance through comparative experiments involving nine famous algorithms. The results demonstrate that ECO either outperforms or shows comparable performance to these algorithms across various problem types.
5. Demonstrates the applicability of the ECO algorithm to several real-world engineering optimization problems, establishing its potential for addressing diverse optimization challenges.

The rest of the paper is organized as follows: Section 2 provides a detailed explanation of our proposed ECO method. Section 3 presents the outcomes of experiments performed on several benchmark functions and real-life issues. Finally, Section 4 concludes the paper and suggests directions for future research.

2 The educational competition optimizer (ECO)

This section elucidates the overall background of ECO and formulates the optimization models.

2.1 Inspiration

Competition in education has become a prominent and contentious issue in contemporary society. As students continuously strive to enhance their abilities and fulfill the stringent admission criteria of educational institutions, the pursuit of higher education has become a relentless quest [46-49]. This pursuit mirrors a fundamental aspect of optimization problems: navigating a vast and complex search space to find the optimal solution. As the level of education rises, the intensity of educational competition increases accordingly. The ECO algorithm continuously retains the elite by simulating this competitive advancement, aligning with the principles of greedy selection and balanced exploration and exploitation in optimization algorithms. This approach not only justifies the methodology but also validates the algorithm's design.

Drawing inspiration from this educational competition, the concept of the educational competition optimizer emerged. This innovative approach offers a fresh perspective on metaheuristic algorithms by metaphorically connecting education and optimization. Consequently, it opens new avenues for devising improved strategies to tackle demanding real-world challenges.

In the primary school stage, characterized by $t \equiv 1(mod\ 3)$, schools select their optimal educational locations based on the population's average location. Students, in turn, compete by aiming for the closest school as their target (approach). In the middle school stage, when $t \equiv 2(mod\ 3)$, the number of schools decreases, prompting schools to consider the best educational location, factoring in both the population's mean position and the best position. Students continue to compete for the nearest school (proximity). Finally, in the high school stage, when $t \equiv 0(mod\ 3)$, schools exercise more careful consideration. They now consider the population's mean, best, and worst positions to determine their educational location. With only one school as their option, students strive to compete for this singular goal (proximity).

2.2 Population initialization

Given that the absence of education leads to societal chaos, we employ logistic chaos mapping to simulate this phenomenon. The initialization formula for logistic chaos mapping, taking into account a population size of N, maximum iterations of Max_{iter}, and search space boundaries of lb (lower bound) and ub (upper bound), can be expressed as:

$$x_i = \alpha \cdot x_{i-1} \cdot (1 - x_{i-1}), 0 \le x_0 \le 1, i = 1, 2, \cdots, N, \alpha = 4 \quad (1)$$

where x_i represents the i^{th} iteration value and x_{i-1} represents the previous iteration value. Map the chaotic value, x_i, to the search space:

$$X_i = lb + (ub - lb) \cdot x_i \quad (2)$$

2.3 Mathematical model of ECO

The ECO algorithm is designed to simulate the dynamics of educational competition, capturing the varying competitive strategies witnessed at different stages: primary school stages, middle school stages, and high school stages. As the competitive pressure heightens and the number of available schools decreases, the optimization process of ECO can be outlined in three steps. By adhering to these conditions, the ECO algorithm smoothly transitions from the exploration step to the exploitation step, relying on an enriched search strategy. We mathematically model the educational competition process as an optimization paradigm to identify the best solution while adhering to specific constraints. The mathematical model of ECO is proposed as follows.

2.3.1Stage 1: primary school stage

During the elementary grades, schools determine suitable teaching locations by considering the average location of the population. On the other hand, students set their individual goals based on the proximity of their neighborhood school. At each iteration, the top 20% of the population, ranked based on their fitness, is categorized as schools, while the remaining 80% constitutes the students. It is important to note that this assignment of roles to individuals such as schools or students can change dynamically throughout the iterations. w is the adaptive step size. **Fig. 1** visually illustrates the behavioral strategies both schools and students adopt at the primary school stage. Primary school students often opt for schools near their residences, considering factors like safety and convenience. In turn, educational institutions often adapt their locations to accommodate the average proximity of their student body, facilitating accessibility and attendance. The mathematical representation of this behavior is denoted by Eq. (3) and Eq. (4).

$$Schools: X_i^{t+1} = X_i^t + w \cdot (X_{imean}^t - X_i^t) \cdot Levy(dim) \quad (3)$$

$$Students: X_i^{t+1} = X_i^t + w \cdot (close(X_i^t) - X_i^t) \cdot randn \quad (4)$$

$$w = 0.1ln\,(2 - \frac{t}{Max_{iter}}) \quad (5)$$

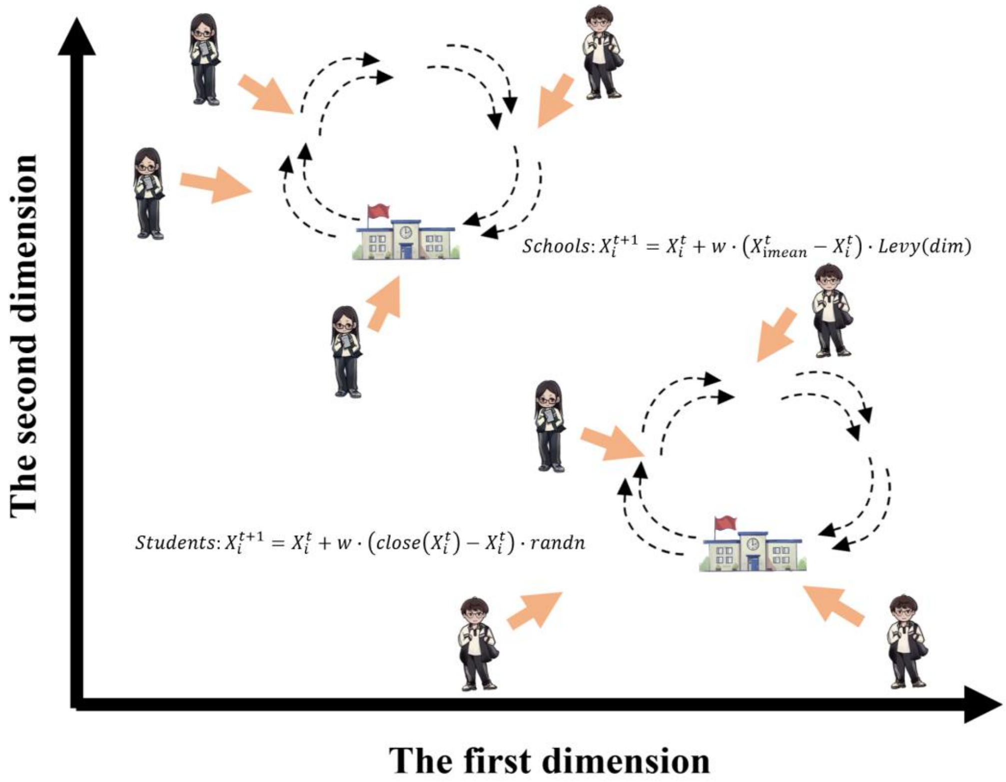


Fig. 1 The behavior at the primary school stage

In Eq. (3) and Eq. (4), X_i^t denotes the current position, while X_i^{t+1} signifies the position of the subsequent update. X_{imean}^t represents the average position of each element of the vector for the i[th] school in the t[th] round of iteration, and $Levy(D)$ denotes the Levy distribution. $close(X)$ indicates the location of the school closest to X. $Randn$ represents a random variable following a normal distribution. The pertinent parameters and functions can be further elucidated as follows:

Average vector position X_{imean}^t & **Average position** X_{mean}^t: X_{imean}^t represents the average position of each element of the vector for the i[th] school in the t[th] round of iteration. X_{mean}^t denotes the average position of the current swarm, denoted as X_{mean}^t. They are calculated as shown in Eq. (6). Where X_{kt} denotes the kth element in the vector X_i^t.

$$\begin{cases} X_{mean}^t = \frac{1}{dim}\sum_{k=1}^{dim} X_{kt} \\ X_{mean}^t = \frac{1}{N}\sum_{k=1}^{N} X_k^t \end{cases} \tag{6}$$

Levy distribution: The rule for the Levy distribution is represented in Eq. (7), where γ is assigned the value of 1.5.

$$\begin{cases} Levy(dim) = \frac{\mu \cdot \sigma}{|v|^{\frac{1}{\gamma}}} \\ \mu \sim N(0, dim) \\ v \sim N(0, dim) \\ \sigma = \left(\frac{\Gamma(1+\gamma)\cdot \sin\left(\frac{\pi\gamma}{2}\right)}{\Gamma\left(\frac{1+\gamma}{2}\right)\cdot\gamma\cdot 2^{\frac{1+\gamma}{2}}}\right)^{\gamma+1} \end{cases} \tag{7}$$

2.3.2 Stage 2: middle school stage

Schools adopt a more sophisticated approach to choosing their teaching location during the middle school stage. They consider a combination of the average and optimal population locations. Similarly, students at this level set their personal goals based on the proximity of neighboring schools. In each iteration, the top 10% of the population, ranked by their fitness, takes on the role of schools, while the remaining 90% constitutes students.

As middle school academic pressure gradually increases, students' patience in learning is denoted by P. Students are further categorized into two groups based on whether they are academically gifted or not. The judgmental threshold H is set at 0.5 for this classification. For academically gifted students, their motivation to learn is represented by E, while those who are not academically talented have a fixed motivation value of $E = 1$. w is the adaptive step size. **Fig. 2** visually presents the behavioral strategies both schools and students adopt at the middle school stage. Like elementary school, the competition among students for better educational resources intensifies. The mathematical representation of these behaviors is expressed by Eq. (8) - Eq. (11).

$$P = 4 \cdot randn \cdot (1 - \frac{i}{Max_{iter}}) \tag{8}$$

$$E = \frac{\pi}{P} \cdot \frac{i}{Max_{iter}} \tag{9}$$

$$Schools: X_i^{t+1} = X_i^t + (X_{best}^t - X_{mean}^t) \cdot exp(\frac{i}{Max_{iter}} - 1) \cdot Levy(dim) \tag{10}$$

$$Students: X_i^{t+1} = \begin{cases} X_i^t - w \cdot close(X_i^t) - P \cdot (E \cdot w \cdot close(X_i^t) - X_i^t), R_1 < H \\ X_i^t - w \cdot close(X_i^t) - P \cdot (w \cdot close(X_i^t) - X_i^t), R_1 \geq H \end{cases} \tag{11}$$

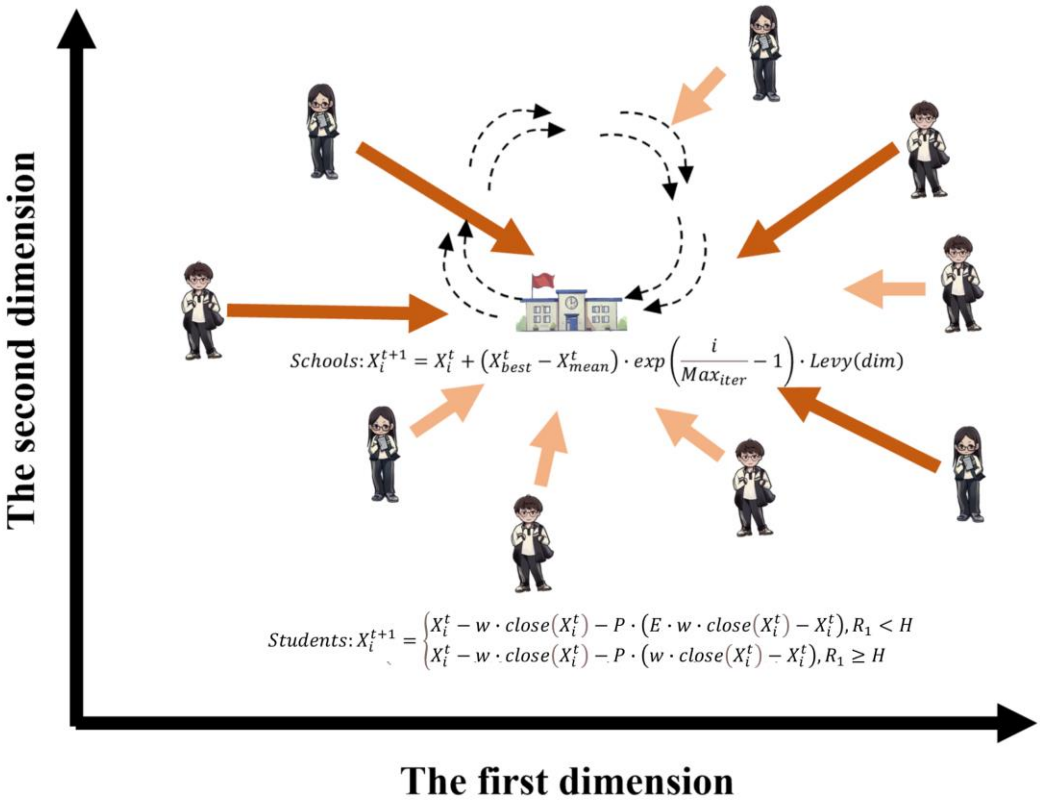


Fig. 2 The behavior at the middle school stage

The talent values of different students are simulated using the random number R_1, which takes on a value within the range of [0, 1].

2.3.3 Stage 3: high school stage

At the high school level, schools adopt a meticulous approach to selecting their teaching locations. They consider not only the average population location but also the best and worst locations within their population. This comprehensive assessment helps them make informed decisions about their educational location. In contrast, all students converge toward the current best location, which is identified as the best high school location. The optimization process motivates every student to strive for admission to this best high school. During each iteration, the top 10% of the population, determined by their fitness, are designated schools, while the remaining 90% continue as students. **Fig. 3** provides a visual representation of the behavioral strategies adopted by both schools and students at the high school level. High schools adapt their locations based on student demographics while students vie for superior educational opportunities, transcending geographical constraints in their pursuit of excellence. Eq. (12) and Eq. (13) represent the mathematical expressions for this behavior.

$$Schools: X_i^{t+1} = X_i^t + \left(X_{best}^t - X_i^t\right) \cdot randn - (X_{best}^t - X_i^t) \cdot randn \tag{12}$$

$$Students: X_i^{t+1} = \begin{cases} X_{best}^t - P \cdot (E \cdot X_{best}^t - X_i^t), R_2 < H \\ X_{best}^t - P \cdot (X_{best}^t - X_i^t), R_2 \geq H \end{cases} \tag{13}$$

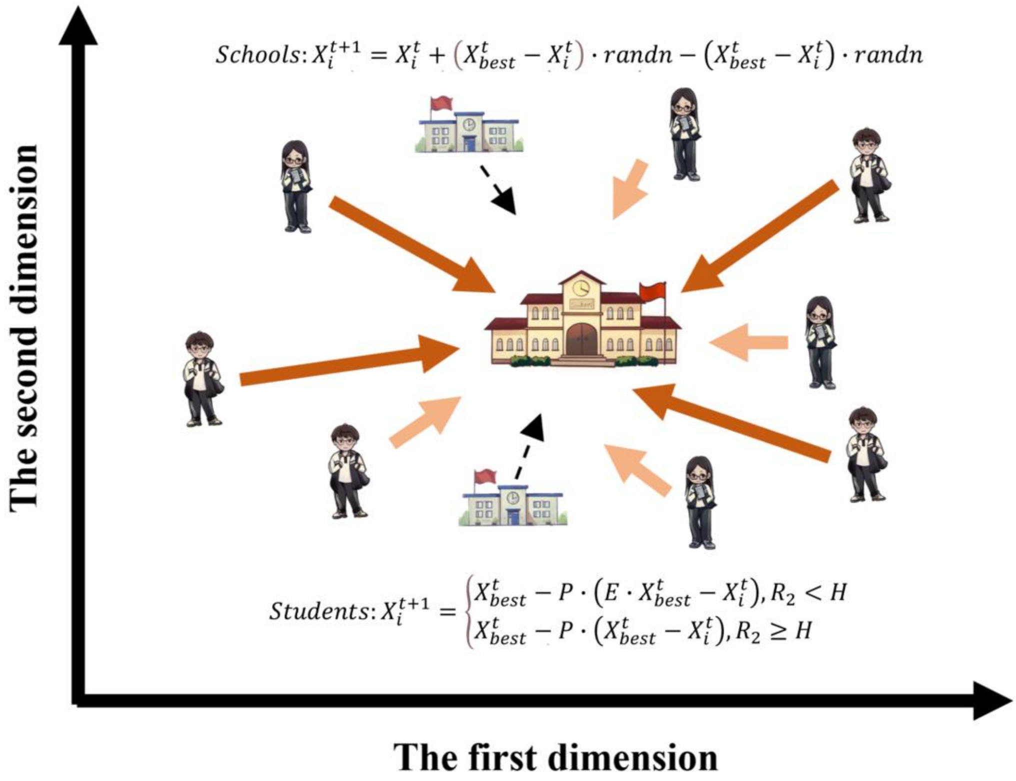


Fig. 3 The behavior at the high school stage

The talents of individual students are represented by a random number denoted as R_2, which falls within the range of [0, 1].

2.4 Pseudo-code of the ECO algorithm

Algorithm 1: Pseudo-code of the ECO algorithm

```
Initialize the ECO parameters
Initialize the solutions' positions randomly (Logistic Chaos Mapping)
For i = 1:Max_iter do
    Calculate the fitness function
    Find the best position and worst position
    Calculate R1, R2, P, E
    For j = 1:N do
        Stage 1: Primary school competition
        If mod(i, 3) == 1 Then
            If j = 1:G1Number Then
                Update schools position by Eq. (3)
            Elseif j = G1Number+1:N Then
                Update students position by Eq. (4)
            End
        Stage 2: middle school competition
        Elseif mod(i, 3) == 2 Then
            If j = 1:G2Number Then
                Update schools position by Eq. (10)
            Elseif j = G2Number+1:N Then
                Update students position by Eq. (11)
            End
        Stage 3:High school competition
        Elseif mod(i, 3) == 0 Then
            If j = 1:G2Number Then
```

Update schools position by Eq. (12)
If j = G2Number+1:N **Then**
Update students position by Eq. (13)
End
End
If $X_i^{t+1} > X_i^t$ **Then**
Select the optimal solution using the positive greedy selection mechanism
End
End
Return the best solution
End

In ECO, the optimization process commences with the random generation of a predetermined set of candidate solutions, known as the population. Through iterative trajectories, ECO's search strategy explores regions proximate to the optimal solution or where the best solution has been identified. Each solution dynamically updates its position based on the best solution attained during ECO's optimization process. ECO places significant emphasis on maintaining a balance between its search strategies: exploration and exploitation. Six distinct exploration and exploitation search strategies are introduced to achieve this balance, involving three phases of interaction between schools and students at different educational levels.

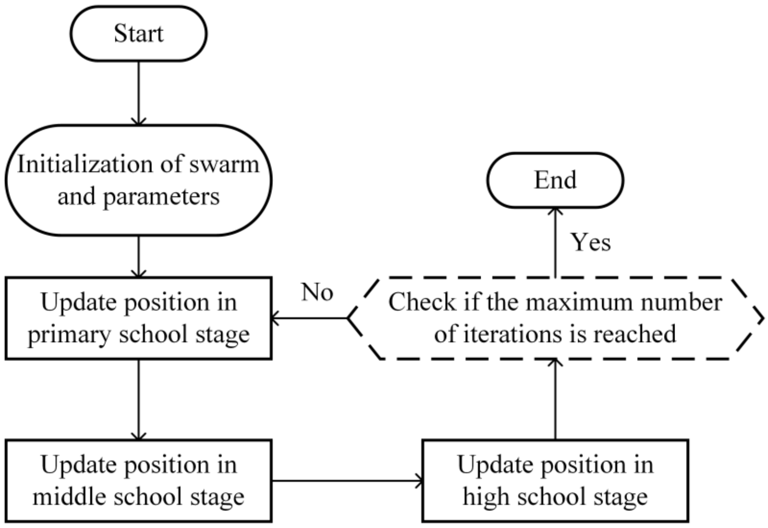


Fig. 4 Flowchart of ECO algorithm

The search process in ECO continues until it meets the predetermined termination criterion. The full architecture of the algorithm is detailed through pseudo-code in Algorithm 1 and illustrated in Fig. 4, providing a thorough walkthrough of the entire optimization process, including its iterative stages and search tactics. ECO leverages the strengths of both exploration and exploitation phases, ensuring a thorough examination of the search space and efficient convergence to optimal solutions.

2.5 Computational complexity of ECO

In this section, we provide an overview of the overall computational complexity associated with the ECO approach. The computational burden of ECO primarily hinges on three key elements: the initialization of solutions, the computation of fitness functions, and the solution update mechanism. Let us consider N as the count of solutions and $O(N)$ as the computational complexity associated with the initialization of these solutions. The computational complexity of the updating processes is $O(T \times N) + O(T \times N \times dim) + O(T \times N \times logN)$, which consists of exploring for the best positions and updating the positions of all solutions, where the total number of iterations is called T and the dimension size of the given problem is called dim.

3 Results and discussion

We assess the efficacy of ECO algorithm by subjecting it to rigorous testing across 23 classical benchmark functions [44], 10 CEC2021 test functions [45], and 6 real-world engineering problems spanning various domains. Subsequently, we conduct a comprehensive comparative analysis by juxtaposing the performance results of ECO against those of nine established metaheuristic algorithms documented in the existing literature, including Ant Lion Optimizer (ALO) [50], Grey Wolf Optimizer (GWO) [2], Whale Optimization Algorithm (WOA) [32], Salp Swarm Algorithm (SSA) [51], Arithmetic Optimization Algorithm (AOA) [52], Harris Hawks Optimization (HHO) [29], Sine Cosine Algorithm (SCA) [53], Multi-Verse Optimizer (MVO) [37], Remora Optimization Algorithm (ROA) [54].

Algorithms	Name of parameters	Value of parameters
ALO	I ratio	10
	w	2-6
GWO	Convergence parameter a	Linear reduction from 2 to 0
WOA	α	Decreased from 2 to 0
	b	2
SSA	v_0	0
AOA	α	5
	μ	0.05
HHO	E_0	[-1,1]
SCA	-	-
MVO	WEP_{max}	1
	WEP_{min}	0.2
ROA	-	-

The algorithms utilized and their specific control parameters are outlined in **Table 1**. We executed these algorithms on both classical and CEC2021 test functions as well as engineering design problems, utilizing MATLAB R2023a. For each algorithm, we conducted 30 independent runs. To assess the quality of the obtained solutions, we employed five performance indicators: best, worst, average, standard deviation (STD), and median values. These indicators were used to showcase the outcomes achieved by the ECO approach.

Researchers frequently utilize a set of 23 classical test functions to assess the performance and capabilities of optimization algorithms. These test functions have been widely employed in numerous optimization algorithm studies. The 23 classical test functions are categorized into three types: single-peak, multi-peak, and fixed-dimension multi-peak functions. In **Table A1 - Table A3**, these functions are delineated alongside their specific details, including function types, search ranges, and theoretical optimal values.

Table 1 Parameter settings

To highlight the effectiveness of the ECO algorithm, we specifically chose to evaluate it using the CEC2021 test functions. These functions exhibit a wide array of characteristics, including unimodal, basic, hybrid, and composite functions. For a deeper understanding of these selected functions, detailed information is provided in **Table A4**.

Table A1 Unimodal benchmark functions

Function	Dim	Range	Shift position	f_{min}
$F_1(x)=\sum_{i=1}^{n} x_i^2$	30	$[-100,100]$	$[-30,-30,\cdots,-30]$	0
$F_2(x)=\sum_{i=1}^{n} \lvert x_i\rvert+\prod_{i=1}^{n} \lvert x_i\rvert$	30	$[-10,10]$	$[-3,-3,\cdots,-3]$	0
$F_3(x)=\sum_{i=1}^{n}(\sum_{j=1}^{n} x_j)^2$	30	$[-100,100]$	$[-30,-30,\cdots,-30]$	0
$F_4(x)=\sum_{i=1}^{n}(\sum_{j=1}^{i} x_j)^2$	30	$[-100,100]$	$[-30,-30,\cdots,-30]$	0
$F_5(x)=\sum_{i=1}^{n-1}\left[100(x_{i+1}-x_i^2)^2+(x_i-1)^2\right]$	30	$[-30,30]$	$[-15,-15,\cdots,-15]$	0

$F_6(x) = \sum_{i=1}^{n} (\lvert x_i + 0.5 \rvert)^2$	30	$[-100,100]$	$[-750,-750,\cdots,-750]$	0
$F_7(x) = \sum_{i=1}^{n} ix^4 + random[0,1)$	30	$[-1.28,1.28]$	$[-0.25,-0.25,\cdots,-0.25]$	0

Table A2 Multimodal benchmark functions.

Function	Dim	Range	Shift position	f_{min}
$F_8(x) = \sum_{i=1}^{n} -x_i sin\,(\sqrt{\lvert x_i \rvert})$	30	$[-500,500]$	$[-300,\cdots,-300]$	-12569.5
$F_9(x) = \sum_{i=1}^{n} \left[x_i^2 - 10\,cos(2\pi x_i) + 10\right]$	30	$[-5.12,5.12]$	$[-2,-2,\cdots,-2]$	0
$F_{10}(x) = -20exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 + e$	30	$[-32,32]$		0
$F_{11}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} cos\left(\frac{x_i}{\sqrt{(i)}}\right) + 1$	30	$[-600,600]$	$[-400,\cdots,-400]$	0
$F_{12}(x) = \frac{\pi}{n}\left\{10sin(\pi y_i) + \sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10sin^2(\pi y_{i+1})] + (y_n - 1)^2\right\} + \sum_{i=1}^{n} u(x_i, 10,100,4)$ $y_i = 1 + \frac{x_i + 1}{4}$ $u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m x_i > a \\ 0 - a < x_i < a \\ k(-x_i - a)^m x_i < -a \end{cases}$	30	$[-50,50]$	$[-30,-30,\cdots,-3$	0
$F_{13}(x) = 0.1\left\{sin^2(3\pi x_1) + \sum_{i=1}^{n1}(x_i - 1)^2[1 + sin^2(3\pi x_i + 1)] + (x_n - 1)^2[1 + sin^2(2\pi x_n)]\right\} + \sum_{i=1}^{n} u(x_i, 5,100,4)$	30	$[-50,50]$	$[-100,\cdots,-100]$	0

Table A3 Fixed-dimension multimodal benchmark functions.

Function	Dim	Range	Shift position	f_{min}
$F_{14}(x) = \left(\frac{1}{500} + \sum_{j=1}^{25}\frac{1}{j + \sum_{i=1}^{2}(x_i - a_{ij})^6}\right)^{-1}$	2	$[-65,65]$	$[-2,-2,\cdots,-2]$	1

$F_{15}(x)=\sum_{i=1}^{11}\left[a_i-\frac{x_1(b_i^2+b_ix_2)}{b_i^2+b_ix_3+x_4}\right]^2$	4	$[-5,5]$	$[-2,-2,\cdots,-2]$	0.0003075
$F_{16}(x)=4x_1^2-2.1x_1^4+\frac{1}{3}x_1^6+x_1x_2-4x_2^2+4x_2^4$	2	$[-5,5]$	$[-2,-2,\cdots,-2]$	-1.0316285
$F_{17}(x)=\left(x_2-\frac{5.1}{4\pi^2}x_1^2+\frac{5}{\pi}x_1-6\right)+10\left(1-\frac{1}{8\pi}\right)\cos x_1+10$	2	$[-5,5]$	$[-2,-2,\cdots,-2]$	0.398
$F_{18}(x)=[1+(x_1+x_2+1)^2(19-14x_1+3x_1^2-14x_2+16x_1x_2+3x_2^2)]\times[30+(2x_1-3x_2)^2\times(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2)]$	2	$[-2,2]$	$[-2,-2,\cdots,-2]$	3
$F_{19}(x)=-\sum_{i=1}^{4}c_i exp\left(-\sum_{j=1}^{3}a_{ij}(x_j-p_{ij})^2\right)$	3	$[1,3]$	$[-2,-2,\cdots,-2]$	-3.86
$F_{20}(x)=-\sum_{i=1}^{4}c_i exp\left(-\sum_{j=1}^{6}a_{ij}(x_j-p_{ij})^2\right)$	6	$[0,1]$	$[-2,-2,\cdots,-2]$	-3.32
$F_{21}(x)=-\sum_{i=1}^{5}[(X-a_i)(X-a_i)^T+c_i]^{-1}$	4	$[0,10]$	$[-2,-2,\cdots,-2]$	-10.1532
$F_{22}(x)=-\sum_{i=1}^{7}[(X-a_i)(X-a_i)^T+c_i]^{-1}$	4	$[0,10]$	$[-2,-2,\cdots,-2]$	-10.4028
$F_{23}(x)=-\sum_{i=1}^{1}[(X-a_i)(X-a_i)^T+c_i]^{-1}$	4	$[0,10]$	$[-2,-2,\cdots,-2]$	-10.536

Table A4 CEC2021 benchmark functions.

	No.	Functions	F_i^*
Unimodal Function	1	Shifted and Rotated Bent Cigar Function	100
Basic Functions	2	Shifted and Rotated Schwefel's Function	1100
	3	Shifted and Rotated Lunacek bi-Rastrigin Function	700
	4	Expanded Rosenbrock's plus Griewangk's Function	1900
Hybrid Functions	5	Hybrid Function 1 (N= 3)	1700
	6	Hybrid Function 2 (N= 4)	1600
	7	Hybrid Function 3 (N= 5)	2100
Composition Functions	8	Composition Function 1(N= 3)	2200
	9	Composition Function 2 (N= 4)	2400
	10	Composition Function 3(N= 5)	2500

Search range: [-100,100]^{dim}

Benchmark datasets serve as widely accepted instruments for assessing the performance of various technologies against established norms [55, 56]. They facilitate the evaluation of distinct computational dimensions, thereby aiding in determining which technology surpasses the rest in multiple fields [57, 58]. By evaluating the algorithm's performance across these 33 test functions, we can gain insights into its effectiveness and efficiency in solving optimization problems.

3.1 Parameter sensitivity analysis

Establishing the optimal population size is paramount for a developed algorithm. In our evaluation of optimization algorithms, we employed the 23 classical test functions to gauge their performance and capabilities.

Table 2 Results of unimodal benchmark functions (different population)

Function	Item	N=10	N=20	N=30	N=40	N=50	N=60
F1	Best	2.10E-77	5.89E-80	1.38E-76	9.69E-92	4.68E-78	8.69E-89
	Median	4.43E-60	1.23E-64	4.76E-64	1.24E-62	1.49E-61	1.42E-67
	Mean	7.41E-44	1.48E-51	5.27E-46	3.77E-49	3.09E-45	1.41E-54
	Worst	2.22E-42	4.39E-50	1.58E-44	1.13E-47	9.28E-44	3.23E-53
	STD	1.58E-85	6.21E-101	8.06E-90	4.12E-96	2.77E-88	3.46E-107
F2	Best	1.22E-39	3.92E-40	1.02E-41	2.01E-45	6.58E-46	4.96E-42
	Median	5.64E-30	1.83E-31	9.71E-32	2.41E-33	2.29E-35	6.08E-34
	Mean	7.68E-24	8.54E-21	2.24E-27	1.33E-27	2.45E-24	2.51E-28
	Worst	1.55E-22	2.56E-19	5.72E-26	2.71E-26	7.34E-23	6.68E-27
	STD	8.79E-46	2.12E-39	1.05E-52	2.43E-53	1.74E-46	1.44E-54
F3	Best	2.53E-74	2.59E-80	1.63E-71	8.41E-83	1.50E-88	4.57E-84
	Median	1.31E-59	6.82E-61	1.30E-60	1.12E-63	4.40E-66	1.66E-66
	Mean	2.97E-44	1.70E-51	8.43E-46	1.72E-52	2.99E-48	5.19E-50
	Worst	7.92E-43	5.10E-50	2.53E-44	5.17E-51	8.76E-47	1.53E-48
	STD	2.04E-86	8.38E-101	2.06E-89	8.61E-103	2.47E-94	7.49E-98
F4	Best	3.08E-39	8.17E-38	1.31E-43	7.52E-45	1.88E-44	5.47E-43
	Median	3.87E-30	5.62E-30	1.32E-31	4.19E-33	5.66E-34	3.93E-34
	Mean	1.28E-26	7.13E-26	1.31E-27	2.90E-26	2.16E-28	1.53E-24
	Worst	2.86E-25	8.37E-25	3.82E-26	8.22E-25	6.04E-27	2.70E-23
	STD	2.65E-51	4.03E-50	4.68E-53	2.18E-50	1.18E-54	3.22E-47
F5	Best	2.73E+01	2.71E+01	2.67E+01	2.65E+01	2.56E+01	2.58E+01
	Median	2.84E+01	2.77E+01	2.73E+01	2.69E+01	2.67E+01	2.66E+01
	Mean	2.82E+01	2.77E+01	2.73E+01	2.70E+01	2.67E+01	2.66E+01
	Worst	2.87E+01	2.84E+01	2.80E+01	2.77E+01	2.74E+01	2.76E+01
	STD	1.41E-01	7.90E-02	1.28E-01	1.47E-01	1.27E-01	1.48E-01
F6	Best	1.24E-02	1.52E-03	4.72E-04	1.54E-04	3.94E-05	1.59E-05
	Median	6.71E-02	1.00E-02	2.75E-03	1.01E-03	3.53E-04	2.89E-04
	Mean	1.36E-01	1.77E-02	5.42E-03	2.69E-03	7.61E-04	5.27E-04
	Worst	5.91E-01	1.34E-01	5.62E-02	2.15E-02	3.30E-03	4.15E-03
	STD	2.56E-02	6.49E-04	1.03E-04	2.13E-05	7.11E-07	6.93E-07
F7	Best	2.08E-05	1.06E-05	1.06E-05	5.47E-06	4.30E-06	1.97E-05
	Median	5.80E-04	2.59E-04	2.73E-04	1.72E-04	1.16E-04	1.21E-04
	Mean	9.09E-04	3.95E-04	3.42E-04	2.01E-04	1.51E-04	1.50E-04
	Worst	2.97E-03	1.58E-03	9.83E-04	6.10E-04	5.59E-04	5.39E-04
	STD	6.97E-07	1.54E-07	7.17E-08	2.83E-08	1.78E-08	1.06E-08

Table 3 Results of multimodal benchmark functions (different population)

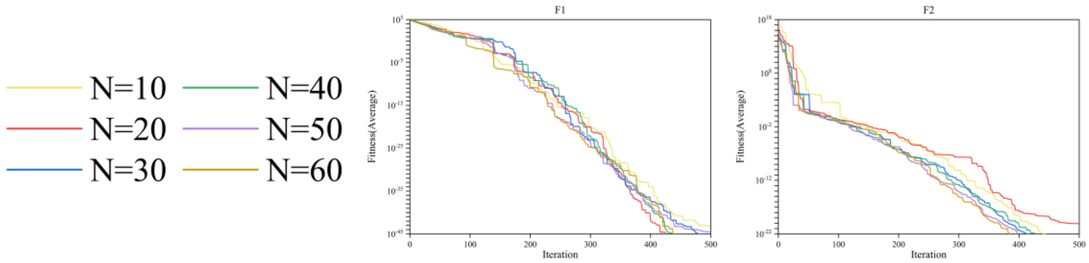
Function	Item	N=10	N=20	N=30	N=40	N=50	N=60
F8	Best	-1.21E+04	-1.21E+04	-1.23E+04	-1.24E+04	-1.24E+04	-1.24E+04
	Median	-1.08E+04	-1.12E+04	-1.14E+04	-1.16E+04	-1.17E+04	-1.17E+04
	Mean	-1.03E+04	-1.11E+04	-1.13E+04	-1.14E+04	-1.14E+04	-1.15E+04
	Worst	-7.51E+03	-8.96E+03	-9.25E+03	-8.68E+03	-8.49E+03	-8.08E+03
	STD	1.54E+06	4.51E+05	7.28E+05	9.98E+05	9.49E+05	7.74E+05
F9	Best	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Median	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Mean	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Worst	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	STD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F10	Best	8.88E-16	8.88E-16	8.88E-16	8.88E-16	8.88E-16	8.88E-16
	Median	8.88E-16	8.88E-16	8.88E-16	8.88E-16	8.88E-16	8.88E-16
	Mean	8.88E-16	8.88E-16	8.88E-16	8.88E-16	8.88E-16	8.88E-16
	Worst	8.88E-16	8.88E-16	8.88E-16	8.88E-16	8.88E-16	8.88E-16
	STD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F11	Best	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Median	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Mean	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Worst	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	STD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F12	Best	2.44E-04	5.85E-05	4.32E-06	3.71E-06	1.54E-06	2.88E-07
	Median	2.63E-03	5.21E-04	8.88E-05	4.07E-05	1.59E-05	5.67E-06
	Mean	5.03E-03	7.11E-04	1.35E-04	6.02E-05	4.11E-05	1.87E-05
	Worst	3.11E-02	3.26E-03	6.94E-04	3.89E-04	3.03E-04	1.01E-04
	STD	4.02E-05	4.40E-07	2.12E-08	5.04E-09	4.09E-09	6.35E-10
F13	Best	4.14E-03	9.56E-04	1.36E-04	8.81E-05	4.14E-05	1.19E-05
	Median	2.45E-01	2.12E-02	1.26E-02	1.16E-02	1.12E-02	6.31E-04
	Mean	6.79E-01	2.48E-01	3.22E-01	1.10E-01	5.20E-02	1.06E-01
	Worst	2.97E+00	2.97E+00	2.97E+00	2.66E+00	1.22E+00	2.97E+00
	STD	1.07E+00	5.36E-01	7.80E-01	2.29E-01	4.72E-02	2.82E-01

Table 4 Results of fixed-dimension multimodal benchmark functions (different population)

Function	Item	N=10	N=20	N=30	N=40	N=50	N=60
F14	Best	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01
	Median	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01
	Mean	2.61E+00	1.49E+00	1.36E+00	1.03E+00	1.78E+00	1.45E+00
	Worst	1.27E+01	5.93E+00	3.97E+00	1.99E+00	1.27E+01	1.27E+01
	STD	6.34E+00	1.35E+00	6.21E-01	3.18E-02	5.29E+00	4.40E+00
F15	Best	3.08E-04	3.07E-04	3.07E-04	3.07E-04	3.07E-04	3.07E-04
	Median	6.42E-04	7.80E-04	7.40E-04	7.62E-04	3.11E-04	3.08E-04
	Mean	2.63E-03	2.07E-03	1.97E-03	1.40E-03	1.29E-03	5.46E-04
	Worst	2.04E-02	2.04E-02	2.04E-02	2.04E-02	2.04E-02	1.22E-03
	STD	3.50E-05	2.40E-05	2.43E-05	1.25E-05	1.27E-05	1.16E-07
F16	Best	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00
	Median	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00
	Mean	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00
	Worst	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00
	STD	6.11E-14	1.76E-16	5.27E-18	2.71E-19	1.70E-24	2.60E-31
F17	Best	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01
	Median	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01
	Mean	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01
	Worst	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01
	STD	1.93E-11	1.05E-13	3.25E-13	2.95E-17	1.03E-15	8.26E-18
F18	Best	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00
	Median	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00
	Mean	4.80E+00	3.00E+00	3.90E+00	3.00E+00	3.00E+00	3.00E+00
	Worst	3.00E+01	3.00E+00	3.00E+01	3.00E+00	3.00E+00	3.00E+00
	STD	4.54E+01	7.15E-27	2.35E+01	1.91E-29	5.36E-30	4.84E-30
F19	Best	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00
	Median	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00
	Mean	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00
	Worst	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00
	STD	1.82E-12	3.46E-21	2.26E-26	6.80E-30	6.25E-30	5.94E-30
F20	Best	-3.32E+00	-3.32E+00	-3.32E+00	-3.32E+00	-3.32E+00	-3.32E+00
	Median	-3.32E+00	-3.20E+00	-3.26E+00	-3.32E+00	-3.20E+00	-3.20E+00
	Mean	-3.27E+00	-3.26E+00	-3.26E+00	-3.27E+00	-3.24E+00	-3.26E+00
	Worst	-3.20E+00	-3.20E+00	-3.20E+00	-3.20E+00	-3.20E+00	-3.20E+00
	STD	3.42E-03	3.52E-03	3.53E-03	3.52E-03	2.97E-03	3.52E-03
F21	Best	-1.02E+01	-1.02E+01	-1.02E+01	-1.02E+01	-1.02E+01	-1.02E+01
	Median	-1.02E+01	-1.02E+01	-1.02E+01	-1.02E+01	-1.02E+01	-1.02E+01
	Mean	-9.30E+00	-8.90E+00	-9.57E+00	-9.98E+00	-9.39E+00	-1.02E+01
	Worst	-2.73E-01	-2.73E-01	-2.73E-01	-5.06E+00	-2.63E+00	-1.02E+01
	STD	6.66E+00	8.21E+00	4.80E+00	8.37E-01	3.91E+00	8.41E-12
F22	Best	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01
	Median	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01
	Mean	-7.74E+00	-8.95E+00	-9.11E+00	-9.38E+00	-9.21E+00	-9.54E+00
	Worst	-2.94E-01	-2.77E+00	-1.84E+00	-2.77E+00	-1.84E+00	-2.77E+00
	STD	1.53E+01	7.12E+00	8.56E+00	6.74E+00	7.34E+00	5.03E+00
F23	Best	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01
	Median	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01
	Mean	-8.76E+00	-9.89E+00	-9.43E+00	-1.03E+01	-9.91E+00	-9.63E+00
	Worst	-3.22E-01	-1.86E+00	-1.68E+00	-3.84E+00	-2.42E+00	-1.86E+00
	STD	1.09E+01	4.04E+00	7.97E+00	1.45E+00	3.75E+00	5.63E+00

Tables 2 to 4 present a detailed analysis of the search results generated by the ECO algorithm following 500 iterations across different population sizes, namely N=10, 20, 30, 40, 50, and 60. These tables provide comprehensive insights into the algorithm's performance, showcasing its efficacy in optimizing solutions.

Fig. 5 depicts the convergence curve displaying the fitness values attained by the ECO algorithm during its quest for optimal solutions spanning from F1 to F23 across a range of population sizes. This graphical representation aids in evaluating and comparing the algorithm's performance across different types of functions and population sizes.



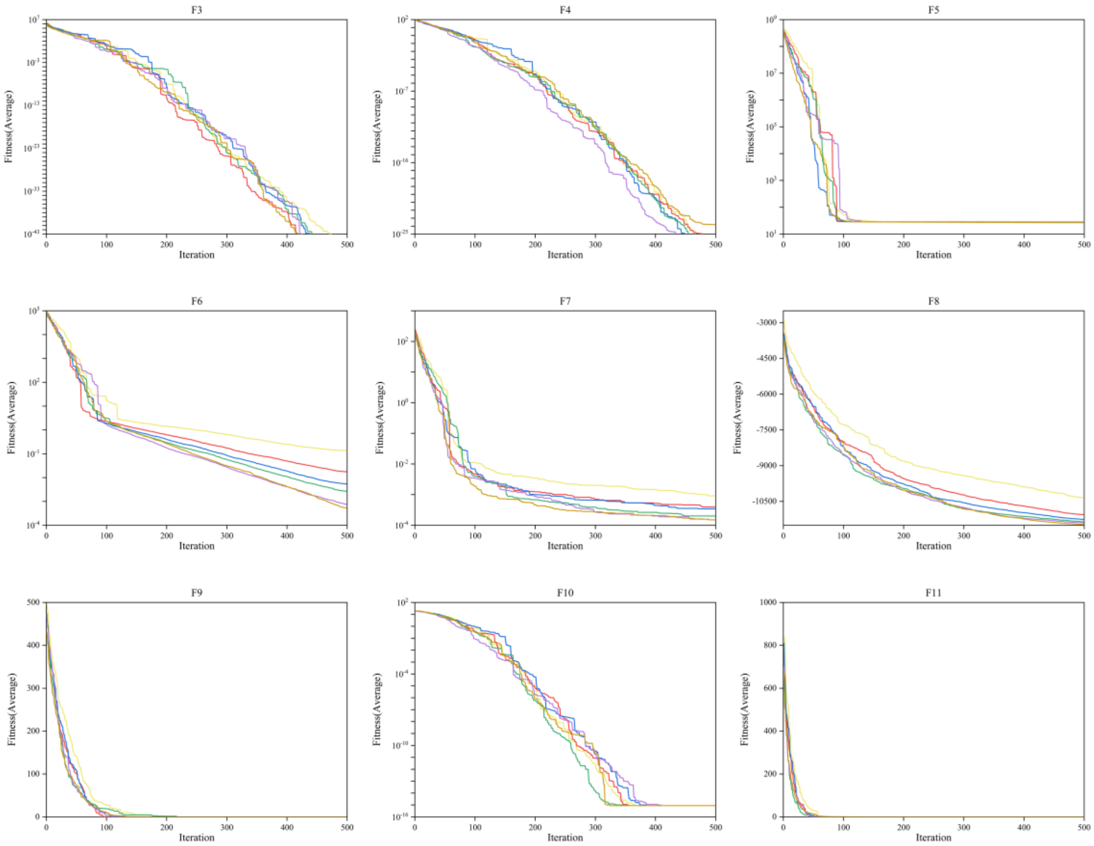
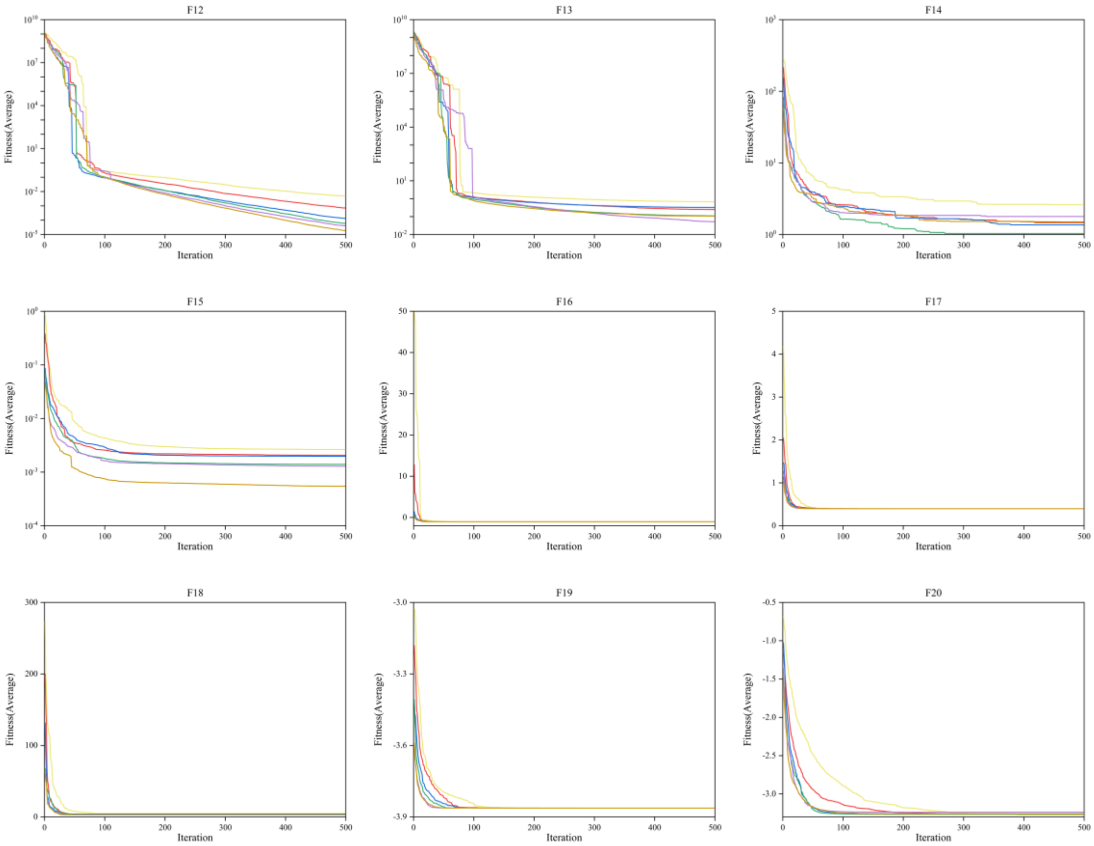


Fig. 5 The influence of the population size



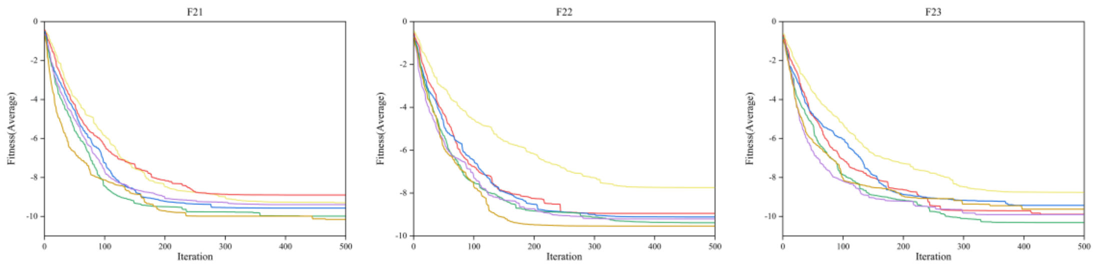


Fig. 5 (Continued)

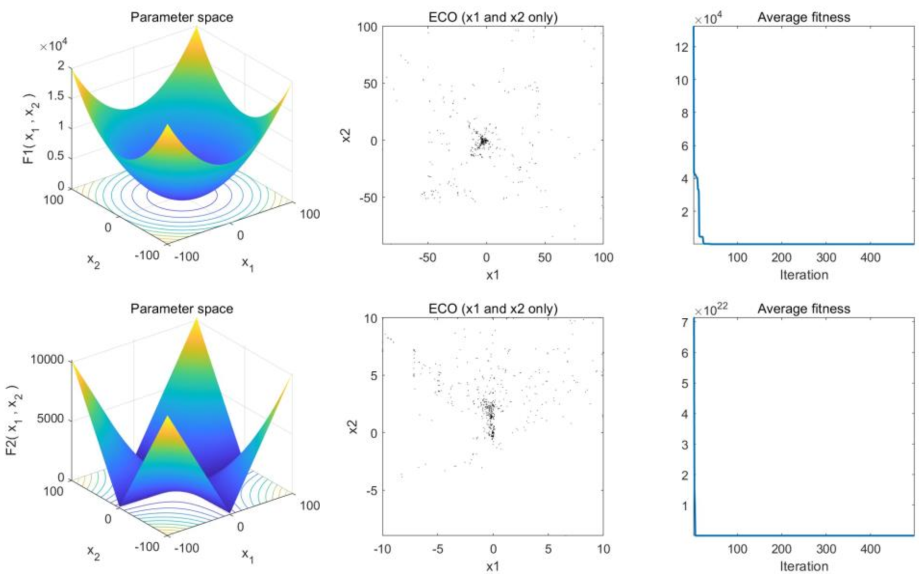
Through our analysis, we observed a positive correlation between the search ability of the ECO algorithm and the population size. The results in **Fig. 5** can be categorized into four groups based on their impact on the outcomes:

1. No effect on the results: Cases such as F11, F17 and F19 show minimal variation, indicating that parameter changes have little influence on the algorithm's performance.
2. Slight effect on the results: F1-F4 demonstrate slight variations due to parameter changes, but the impact is relatively minor.
3. Large effect on the results: F12 and F15 exhibit significant performance variations due to parameter changes.
4. Larger populations lead to worse results: F4 and F23 show that increasing the population size can result in poorer algorithm performance.

Most cases fall into the first two categories, indicating the ECO model's strong robustness. It demonstrates stable and reliable performance across various scenarios and parameter settings. Notably, all the test functions exhibit rapid convergence during the initial iteration phase, thanks to the alternating search dynamics of the three educational competition phases. This underscores ECO's remarkable ability to locate near-optimal solutions swiftly.

Intuitively, larger population sizes generally enhance the search scope and improve the likelihood of discovering an optimal solution. However, substantial populations may significantly prolong computational time. Notably, empirical findings from ECO studies reveal that the model's performance is relatively unaffected by population size variations. In certain test scenarios, like functions F13, F20, and F23, increasing population size actually diminishes the optimization efficacy of the ECO model. To balance computational efficiency and optimization performance, we opt for a population size parameter of N = 40 in the ECO framework.

To affirm the efficacy of the ECO algorithm, we executed experiments and scrutinized its convergence and trajectories. Fig. 6 vividly presents these findings, offering a glimpse into the evolution of search points within the population and the concurrent fluctuations in the average fitness as the ECO algorithm pursues the optimal solution with a population size of N = 40. This visualization in Fig. 6 provides intricate insights into the dynamic interplay between schools and students throughout the optimization process.



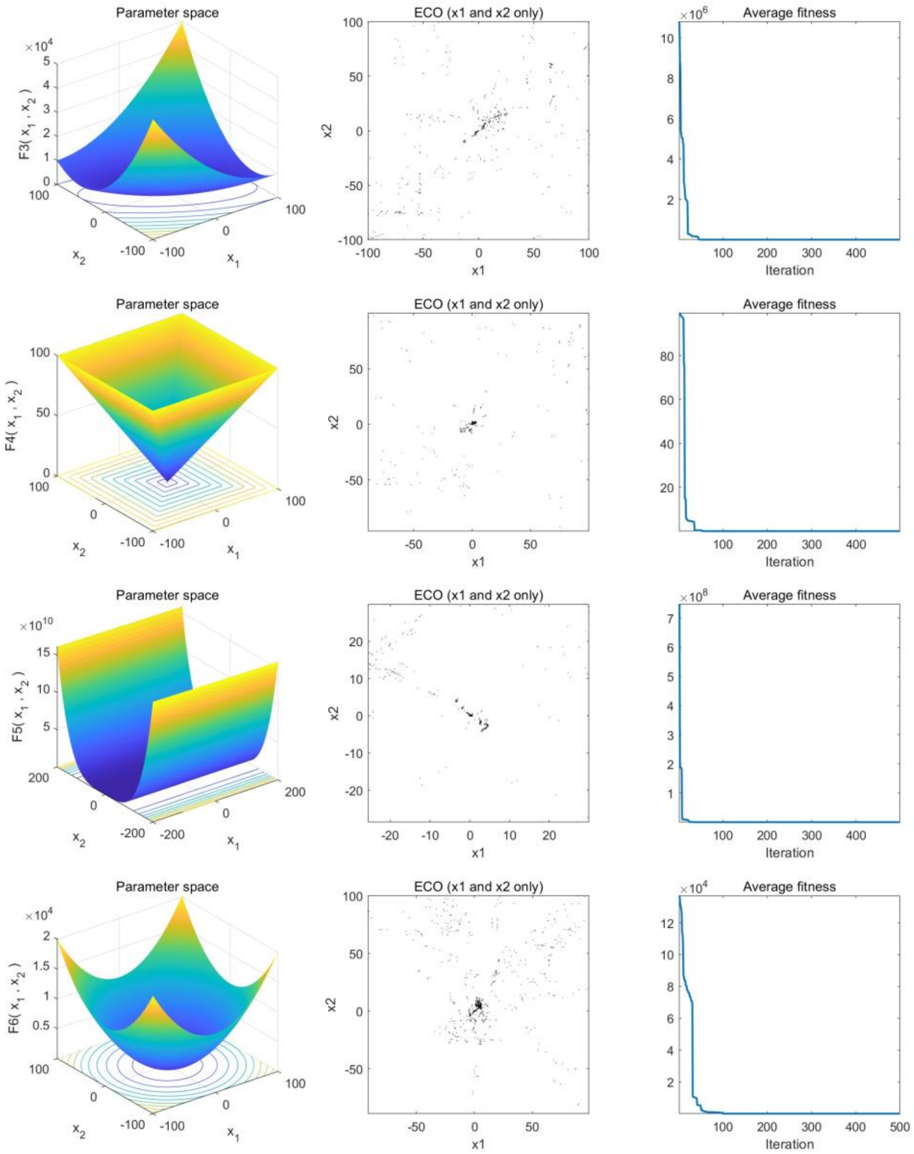
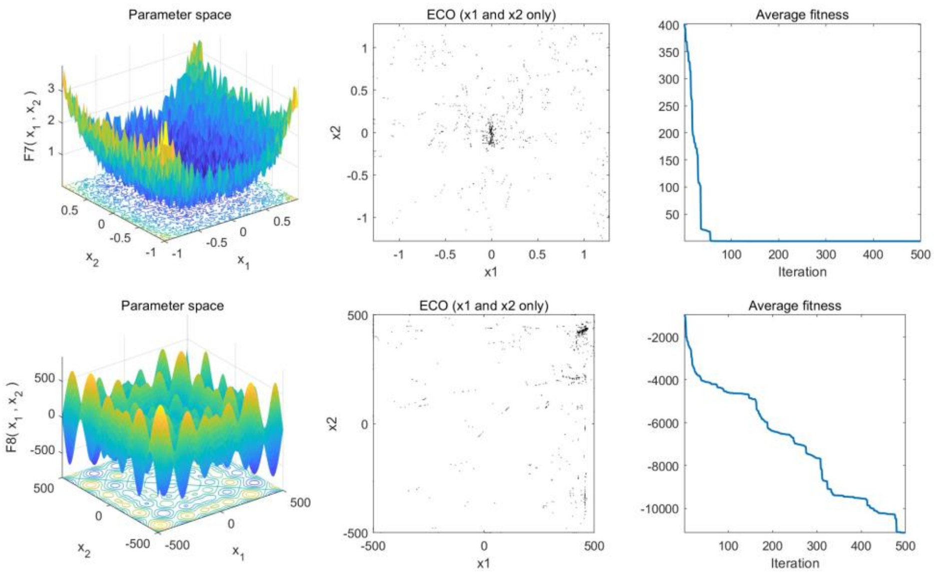


Fig. 6 Qualitative results for the studied problems



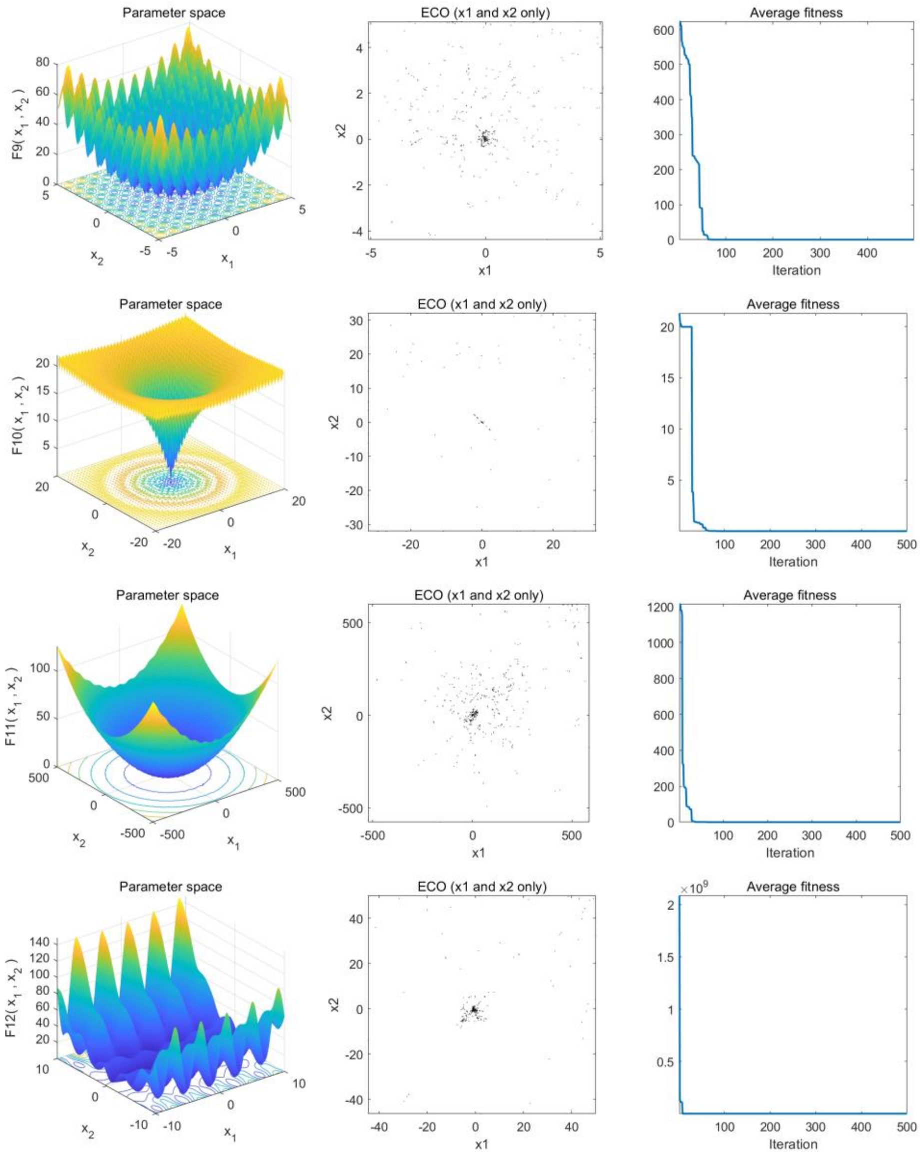
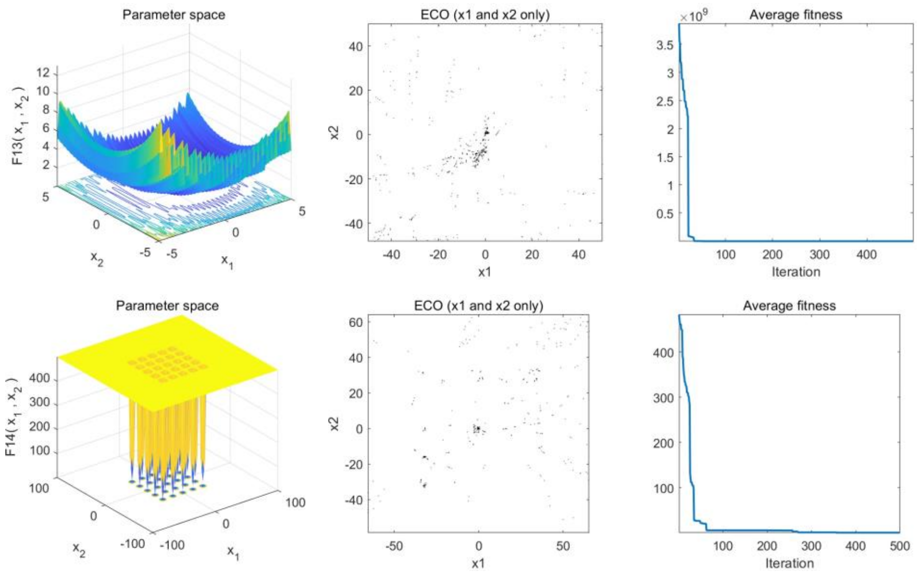


Fig. 6 (Continued)



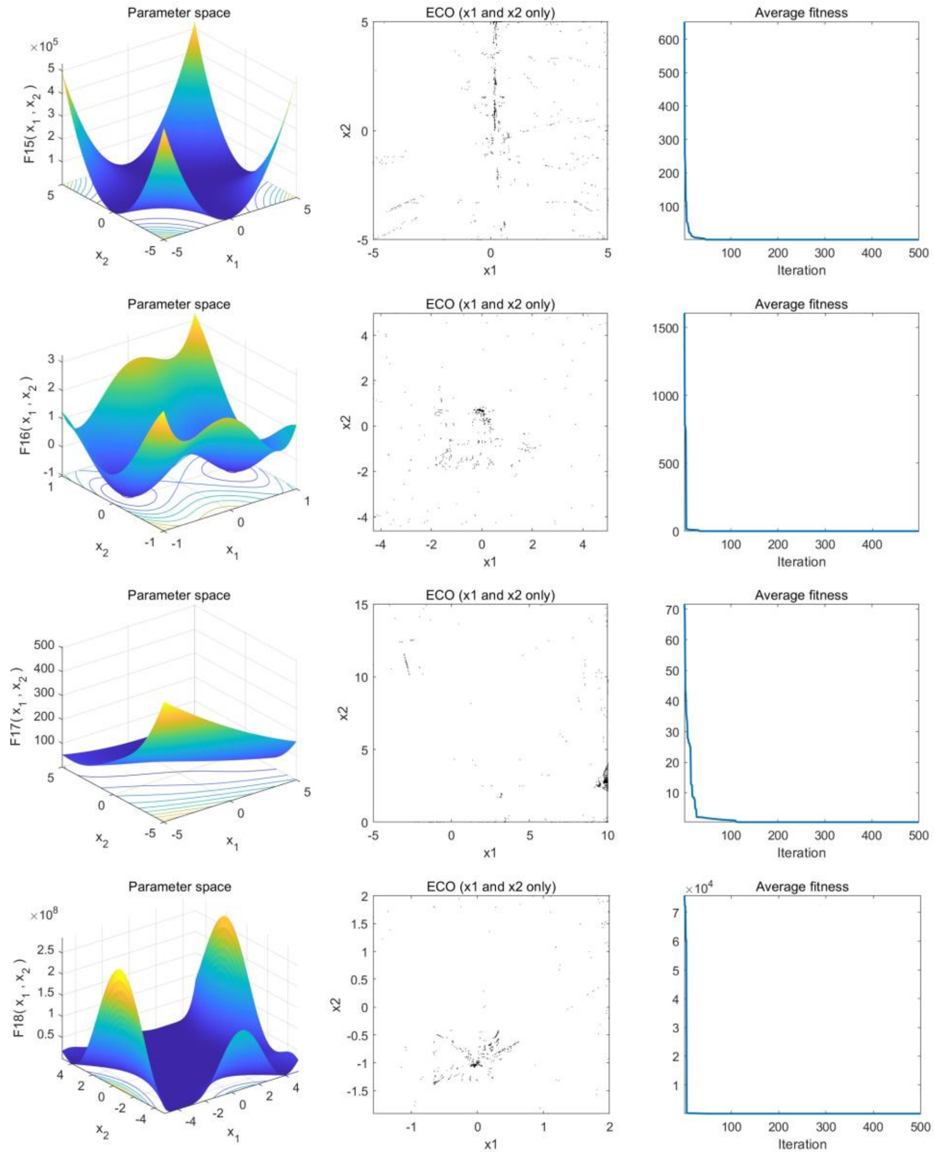
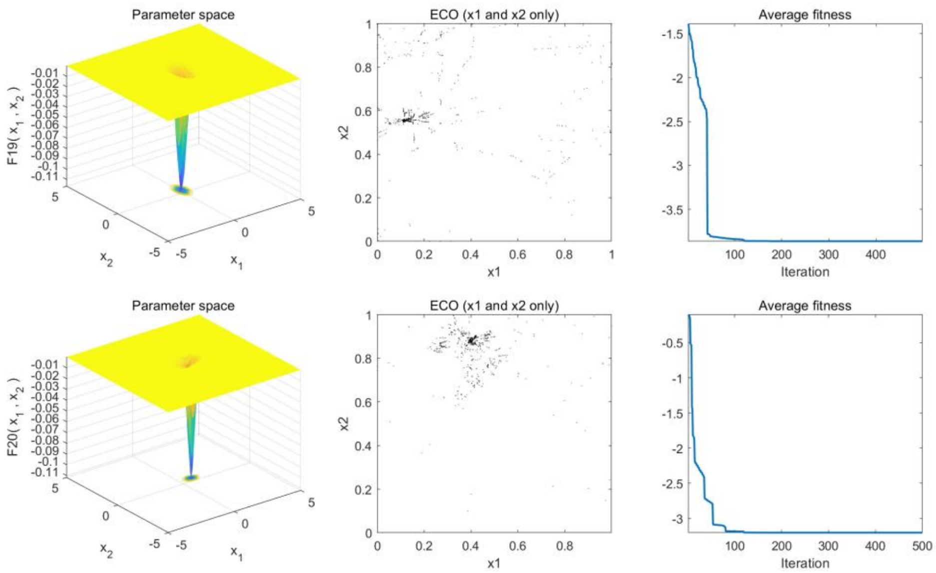


Fig. 6 (Continued)



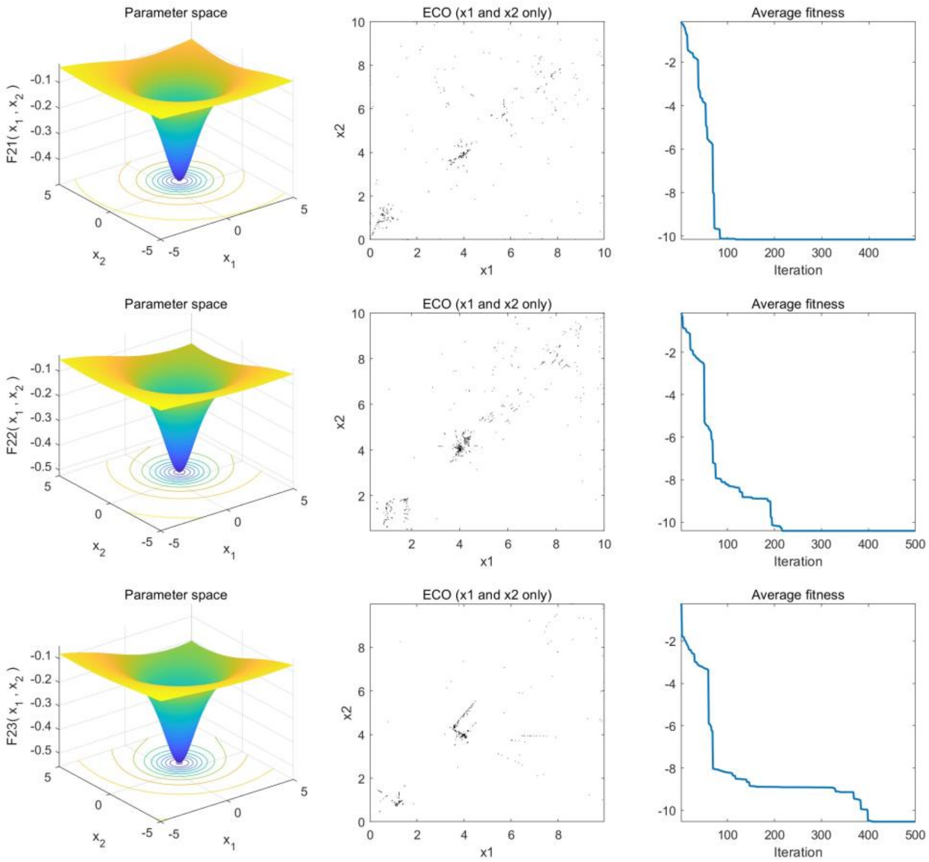
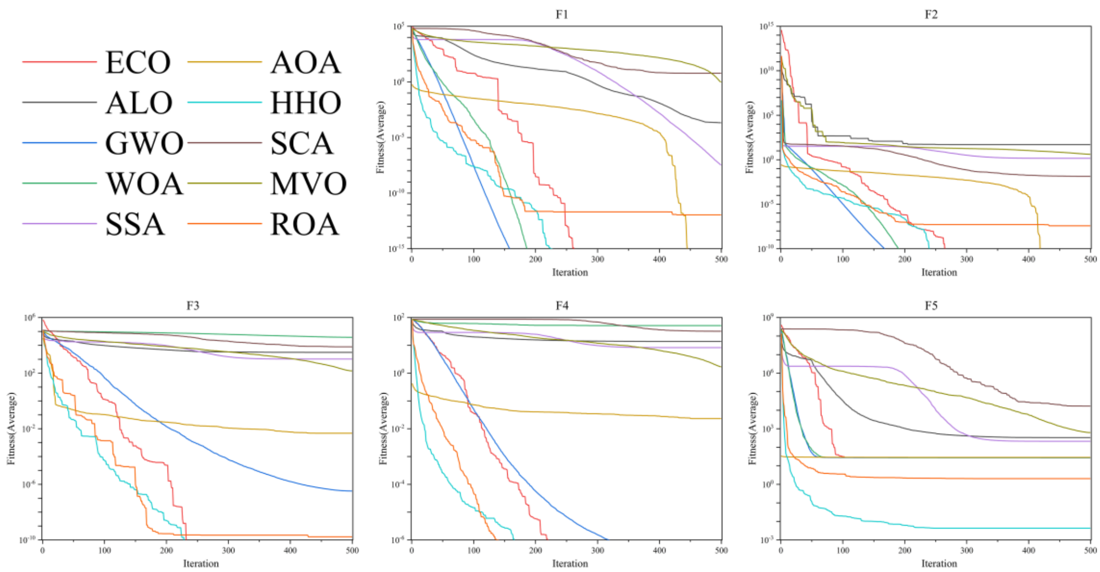


Fig. 6 (Continued)

3.2 Comparison of different algorithms on classical test functions

To rigorously assess and contrast the search capabilities of the ECO algorithm, we meticulously selected nine state-of-the-art algorithms for a comparative analysis on a classical test function. To ensure impartiality in the comparison, all examined algorithms underwent execution with uniform parameters: 500 iterations and a population size of 40, aligning precisely with the settings of ECO.

This meticulous approach empowers us to gauge the relative performance and efficacy of ECO against the chosen algorithms under uniform experimental conditions. Through this meticulously crafted evaluation framework, we can thoroughly scrutinize and discern the search prowess of the algorithms on an equal footing.



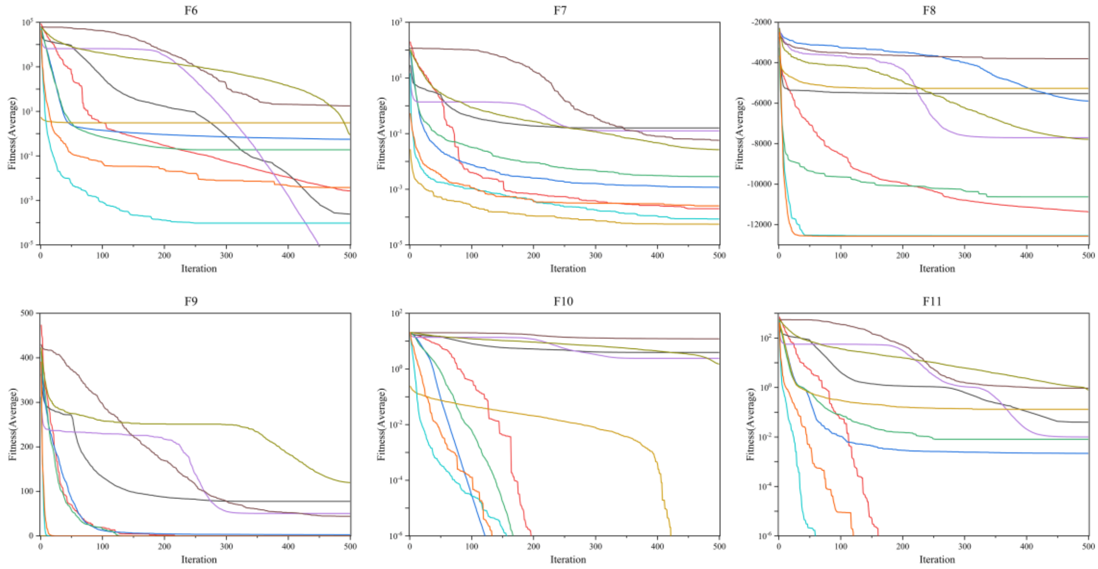


Fig. 7 Comparison of convergence rates for different algorithms

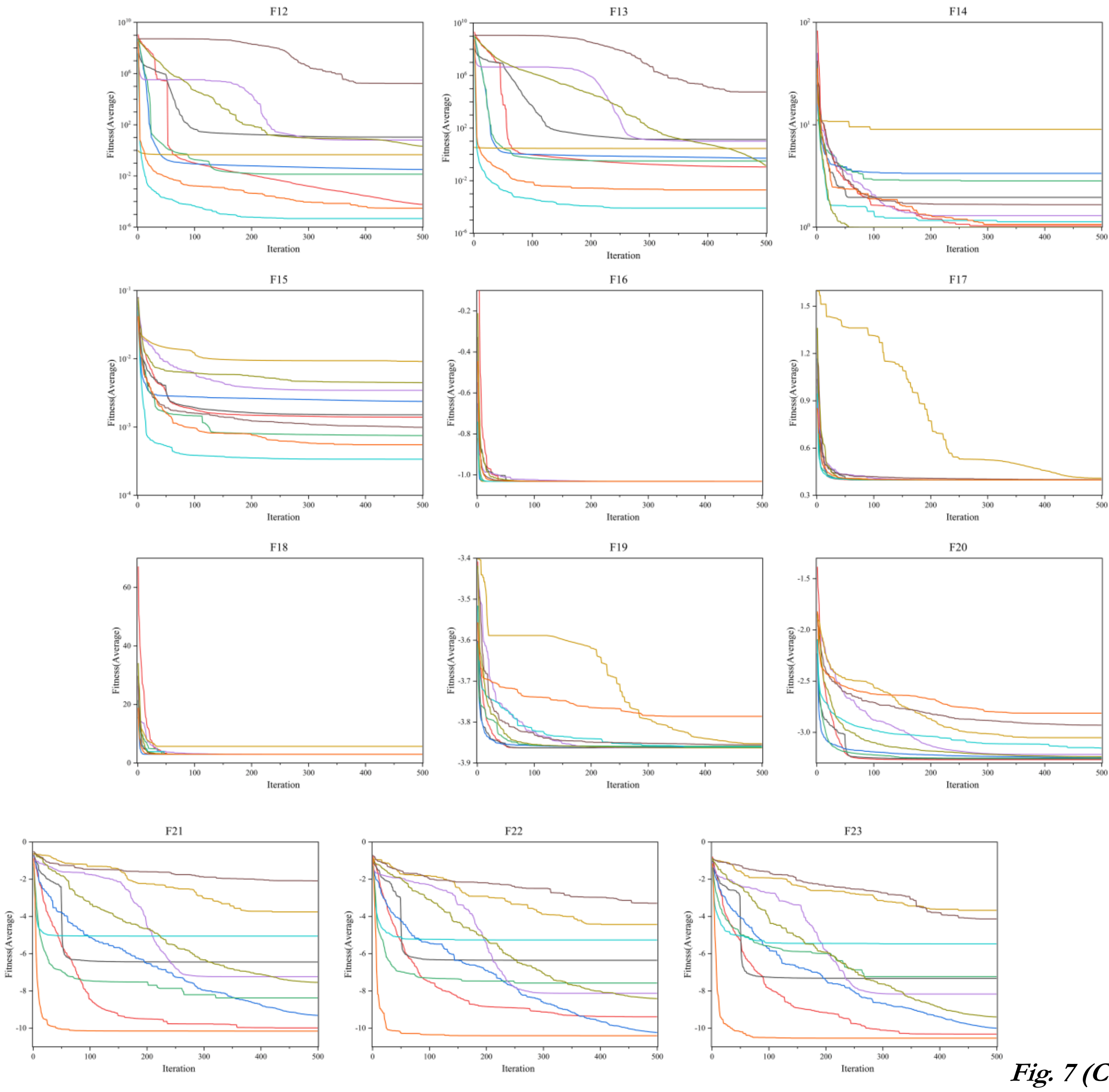


Fig. 7 (Continued)

Table 5 Results of unimodal benchmark functions (different algorithms)

	Item	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
F1	Best	9.69E-92	7.59E-05	2.15E-32	3.73E-91	1.19E-08	5.67E-243	6.67E-116	8.60E-03	4.52E-01	3.66E-34
	Median	1.24E-62	1.87E-04	3.66E-31	4.28E-85	3.05E-08	4.14E-	3.48E-	1.25E+00	9.47E-01	4.13E-21

							132	107			
	Mean	3.77E-49	2.12E-04	5.68E-31	4.10E-79	3.44E-08	7.77E-24	1.88E-95	5.98E+00	9.96E-01	1.11E-12
	Worst	1.13E-47	5.37E-04	2.84E-30	1.22E-77	8.11E-08	2.33E-22	5.60E-94	1.03E+02	1.95E+00	3.34E-11
	STD	4.12E-96	1.16E-08	4.70E-61	4.76E-156	2.52E-16	1.75E-45	1.01E-188	3.39E+02	1.46E-01	3.60E-23
F2	Best	2.01E-45	7.43E-01	3.19E-19	9.57E-58	9.09E-02	0.00E+00	3.63E-61	3.00E-05	3.96E-01	5.89E-16
	Median	2.41E-33	1.87E+01	1.07E-18	2.86E-55	1.32E+00	0.00E+00	2.06E-54	5.88E-03	7.13E-01	1.63E-10
	Mean	1.33E-27	5.00E+01	1.50E-18	4.20E-53	1.45E+00	0.00E+00	2.34E-50	1.38E-02	4.10E+00	3.77E-08
	Worst	2.71E-26	1.22E+02	4.11E-18	6.40E-52	3.71E+00	0.00E+00	6.81E-49	7.00E-02	9.90E+01	7.54E-07
	STD	2.43E-53	2.33E+03	1.16E-36	1.51E-104	8.92E-01	0.00E+00	1.49E-98	3.26E-04	3.11E+02	1.90E-14
F3	Best	8.41E-83	1.28E+03	5.66E-10	1.83E+04	9.13E+01	5.79E-153	1.12E-102	1.26E+03	4.14E+01	1.27E-23
	Median	1.12E-63	3.00E+03	4.54E-08	3.78E+04	7.56E+02	6.96E-43	6.05E-93	7.67E+03	1.30E+02	6.98E-15
	Mean	1.72E-52	3.11E+03	3.33E-07	3.78E+04	1.03E+03	4.76E-03	1.73E-84	7.82E+03	1.46E+02	1.69E-10
	Worst	5.17E-51	6.73E+03	2.74E-06	5.32E+04	3.60E+03	5.31E-02	3.50E-83	1.67E+04	3.23E+02	5.00E-09
	STD	8.61E-103	1.80E+06	4.63E-13	7.32E+07	7.65E+05	1.24E-04	4.25E-167	2.42E+07	4.03E+03	8.05E-19
F4	Best	7.52E-45	7.65E+00	8.86E-09	2.76E-01	3.80E+00	1.40E-84	3.40E-58	1.18E+01	6.48E-01	5.72E-17
	Median	4.19E-33	1.29E+01	6.90E-08	5.40E+01	8.43E+00	2.63E-02	4.15E-54	3.46E+01	1.62E+00	7.30E-11
	Mean	2.90E-26	1.38E+01	1.50E-07	5.13E+01	8.27E+00	2.32E-02	1.66E-50	3.22E+01	1.70E+00	6.56E-09
	Worst	8.22E-25	2.04E+01	9.64E-07	9.22E+01	1.31E+01	4.58E-02	4.25E-49	4.66E+01	3.18E+00	9.96E-08
	STD	2.18E-50	1.07E+01	5.09E-14	6.73E+02	6.86E+00	3.43E-04	5.83E-99	8.85E+01	3.79E-01	4.95E-16
F5	Best	2.65E+01	2.38E+01	2.56E+01	2.68E+01	2.62E+01	2.76E+01	1.93E-05	3.30E+01	3.07E+01	2.73E-06
	Median	2.69E+01	1.15E+02	2.66E+01	2.76E+01	7.89E+01	2.84E+01	2.46E-03	1.88E+03	2.39E+02	2.65E-02
	Mean	2.70E+01	3.34E+02	2.69E+01	2.76E+01	2.12E+02	2.84E+01	4.34E-03	1.67E+04	6.23E+02	2.04E+00
	Worst	2.77E+01	2.10E+03	2.86E+01	2.87E+01	1.28E+03	2.89E+01	1.96E-02	1.33E+05	2.85E+03	2.87E+01
	STD	1.47E-01	2.72E+05	6.98E-01	1.50E-01	9.32E+04	7.47E-02	2.63E-05	9.73E+08	7.41E+05	5.09E+01
F6	Best	1.54E-04	6.39E-05	4.80E-05	4.34E-02	1.42E-08	2.49E+00	1.19E-08	4.11E+00	4.60E-01	1.49E-06
	Median	1.01E-03	2.10E-04	5.02E-01	1.56E-01	3.07E-08	3.09E+00	3.30E-05	7.74E+00	8.44E-01	7.05E-04
	Mean	2.69E-03	2.42E-04	5.58E-01	1.88E-01	3.27E-08	3.05E+00	9.57E-05	1.77E+01	9.20E-01	3.79E-03
	Worst	2.15E-02	6.72E-04	1.00E+00	7.29E-01	8.01E-08	3.47E+00	4.35E-04	1.63E+02	1.38E+00	3.55E-02
	STD	2.13E-05	2.04E-08	8.56E-02	1.90E-02	1.78E-16	6.36E-02	1.50E-08	9.48E+02	6.43E-02	7.30E-05
F7	Best	5.47E-06	6.12E-02	3.29E-04	7.78E-05	4.94E-02	4.47E-07	7.19E-07	7.64E-03	7.17E-03	1.74E-05
	Median	1.72E-04	1.54E-01	1.05E-03	1.44E-03	1.10E-01	4.10E-05	6.17E-05	3.70E-02	2.41E-02	1.61E-04
	Mean	2.01E-04	1.60E-01	1.17E-03	2.86E-03	1.26E-01	5.58E-05	8.56E-05	5.78E-02	2.65E-02	2.53E-04
	Worst	6.10E-04	3.13E-01	2.73E-03	1.69E-02	3.38E-01	1.93E-04	5.14E-04	2.32E-01	5.89E-02	1.24E-03
	STD	2.83E-08	3.40E-03	3.32E-07	1.16E-05	3.27E-03	2.46E-09	9.68E-09	2.49E-03	1.31E-04	7.97E-08

Table 6 Results of multimodal benchmark functions (different algorithms)

Table 7 Results of fixed-dimension multimodal benchmark functions (different algorithms)

	Item	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
F8	Best	-1.24E+04	-6.53E+03	-7.44E+03	-1.26E+04	-9.81E+03	-5.92E+03	-1.26E+04	-4.78E+03	-8.88E+03	-1.26E+04
	Median	-1.16E+04	-5.51E+03	-5.83E+03	-1.11E+04	-7.77E+03	-5.37E+03	-1.26E+04	-3.73E+03	-7.93E+03	-1.26E+04
	Mean	-1.14E+04	-5.52E+03	-5.89E+03	-1.06E+04	-7.70E+03	-5.26E+03	-1.25E+04	-3.80E+03	-7.78E+03	-1.26E+04
	Worst	-8.68E+03	-5.42E+03	-3.46E+03	-6.83E+03	-5.98E+03	-4.45E+03	-1.21E+04	-3.32E+03	-6.55E+03	-1.26E+04
	STD	9.98E+05	3.96E+04	8.11E+05	3.22E+06	6.12E+05	1.66E+05	7.75E+03	1.04E+05	4.35E+05	2.11E-05
F9	Best	0.00E+00	4.38E+01	0.00E+00	0.00E+00	2.39E+01	0.00E+00	0.00E+00	8.79E-03	7.02E+01	0.00E+00
	Median	0.00E+00	7.81E+01	5.02E-01	0.00E+00	4.78E+01	0.00E+00	0.00E+00	3.47E+01	1.14E+02	0.00E+00
	Mean	0.00E+00	7.77E+01	2.74E+00	0.00E+00	5.06E+01	0.00E+00	0.00E+00	4.42E+01	1.20E+02	3.73E-13
	Worst	0.00E+00	1.53E+02	1.61E+01	0.00E+00	9.25E+01	0.00E+00	0.00E+00	1.32E+02	1.99E+02	1.07E-11
	STD	0.00E+00	5.44E+02	1.45E+01	0.00E+00	3.07E+02	0.00E+00	0.00E+00	1.33E+03	8.67E+02	3.71E-24
F10	Best	8.88E-16	1.16E+00	3.95E-14	4.44E-16	1.16E+00	4.44E-16	4.44E-16	2.03E-02	5.25E-01	4.44E-16
	Median	8.88E-16	3.09E+00	5.55E-14	4.00E-15	2.50E+00	4.44E-16	4.44E-16	1.97E+01	1.50E+00	1.57E-11
	Mean	8.88E-16	3.92E+00	5.56E-14	4.47E-15	2.40E+00	4.44E-16	4.44E-16	1.21E+01	1.52E+00	1.13E-09
	Worst	8.88E-16	1.25E+01	6.44E-14	7.55E-15	3.98E+00	4.44E-16	4.44E-16	2.04E+01	2.64E+00	2.03E-08
	STD	0.00E+00	7.83E+00	4.10E-29	6.51E-30	6.19E-01	0.00E+00	0.00E+00	8.63E+01	2.52E-01	1.43E-17
F11	Best	0.00E+00	8.45E-03	0.00E+00	0.00E+00	4.53E-05	7.42E-03	0.00E+00	4.74E-01	6.55E-01	0.00E+00
	Median	0.00E+00	3.53E-02	0.00E+00	0.00E+00	8.41E-03	1.18E-01	0.00E+00	9.07E-01	8.44E-01	0.00E+00
	Mean	0.00E+00	3.98E-02	2.17E-03	8.09E-03	1.00E-02	1.31E-01	0.00E+00	9.40E-01	8.11E-01	1.19E-13
	Worst	0.00E+00	8.93E-02	1.71E-02	1.57E-01	3.72E-02	3.93E-01	0.00E+00	2.50E+00	9.44E-01	2.14E-12
	STD	0.00E+00	5.08E-04	2.48E-05	1.00E-03	9.32E-05	8.69E-03	0.00E+00	1.50E-01	5.27E-03	1.74E-25
F12	Best	3.71E-06	6.39E+00	6.68E-03	2.57E-03	1.75E+00	3.48E-01	5.78E-08	1.20E+00	3.88E-01	1.59E-09
	Median	4.07E-05	9.52E+00	2.68E-02	8.30E-03	6.29E+00	4.63E-01	2.03E-06	1.12E+01	1.99E+00	1.71E-05
	Mean	6.02E-05	1.09E+01	3.23E-02	1.38E-02	6.42E+00	4.58E-01	4.73E-06	1.70E+05	2.15E+00	3.04E-05
	Worst	3.89E-04	2.29E+01	9.12E-02	9.77E-02	1.38E+01	5.58E-01	2.12E-05	4.85E+06	5.30E+00	1.83E-04
	STD	5.04E-09	2.03E+01	2.83E-04	3.10E-04	6.38E+00	2.14E-03	3.66E-11	7.58E+11	1.61E+00	1.68E-09
F13	Best	8.81E-05	1.14E-02	9.83E-02	5.18E-02	6.91E-05	2.57E+00	1.92E-07	2.45E+00	3.86E-02	4.30E-06
	Median	1.16E-02	5.09E+00	5.14E-01	2.78E-01	1.11E+00	2.83E+00	5.37E-05	9.98E+01	1.23E-01	7.23E-04
	Mean	1.10E-01	1.35E+01	5.14E-01	3.15E-01	1.00E+01	2.82E+00	8.22E-05	5.61E+04	1.44E-01	1.97E-03
	Worst	2.66E+00	6.22E+01	9.17E-01	7.31E-01	4.38E+01	2.99E+00	5.33E-04	6.92E+05	3.31E-01	1.29E-02
	STD	2.29E-01	2.62E+02	3.88E-02	3.09E-02	1.48E+02	1.02E-02	1.13E-08	2.79E+10	4.83E-03	1.11E-05

	Item	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
F14	Best	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01
	Median	9.98E-01	1.99E+00	2.98E+00	1.50E+00	9.98E-01	1.12E+01	9.98E-01	9.99E-01	9.98E-01	9.98E-01
	Mean	1.03E+00	1.96E+00	3.35E+00	2.83E+00	1.30E+00	9.03E+00	1.13E+00	1.66E+00	9.98E-01	1.06E+00
	Worst	1.99E+00	6.90E+00	1.27E+01	1.08E+01	3.97E+00	1.27E+01	1.99E+00	2.98E+00	9.98E-01	2.98E+00
	STD	3.18E-02	1.85E+00	1.17E+01	8.71E+00	5.34E-01	1.63E+01	1.14E-01	8.74E-01	1.52E-21	1.27E-01
F15	Best	3.07E-04	3.08E-04	3.08E-04	3.15E-04	4.21E-04	3.56E-04	3.08E-04	3.46E-04	5.68E-04	3.08E-04
	Median	7.62E-04	8.24E-04	3.29E-04	6.47E-04	7.85E-04	2.82E-03	3.31E-04	7.87E-04	7.69E-04	3.45E-04
	Mean	1.40E-03	1.51E-03	2.38E-03	7.52E-04	3.46E-03	9.13E-03	3.38E-04	9.91E-04	4.48E-03	5.52E-04
	Worst	2.04E-02	2.04E-02	2.04E-02	1.86E-03	2.04E-02	3.56E-02	4.01E-04	1.84E-03	2.04E-02	1.69E-03
	STD	1.26E-05	1.23E-05	3.60E-05	2.17E-07	4.40E-05	1.19E-04	7.82E-10	1.48E-07	5.32E-05	1.59E-07
F16	Best	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00
	Median	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00
	Mean	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00
	Worst	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00
	STD	2.71E-19	2.90E-26	2.21E-16	1.26E-19	8.64E-28	1.30E-14	5.07E-21	1.33E-09	6.63E-14	9.84E-10
F17	Best	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01
	Median	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	4.07E-01	3.98E-01	3.99E-01	3.98E-01	3.98E-01
	Mean	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	4.09E-01	3.98E-01	4.00E-01	3.98E-01	3.98E-01
	Worst	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	4.29E-01	3.98E-01	4.09E-01	3.98E-01	4.01E-01
	STD	2.95E-17	6.05E-27	1.62E-11	9.36E-12	5.14E-28	7.57E-05	4.80E-11	7.45E-06	2.28E-13	4.78E-07
F18	Best	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00
	Median	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00
	Mean	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	5.70E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00
	Worst	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+01	3.00E+00	3.00E+00	3.00E+00	3.00E+00
	STD	1.91E-29	1.49E-25	3.71E-10	5.29E-10	7.70E-26	6.56E+01	6.63E-13	9.63E-09	5.22E-12	9.44E-08
F19	Best	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00
	Median	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.85E+00	-3.86E+00	-3.85E+00	-3.86E+00	-3.84E+00
	Mean	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.85E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.79E+00
	Worst	-3.86E+00	-3.86E+00	-3.85E+00	-3.83E+00	-3.86E+00	-3.84E+00	-3.85E+00	-3.85E+00	-3.86E+00	-3.61E+00
	STD	6.80E-30	1.01E-26	5.77E-06	3.33E-05	2.47E-25	1.06E-05	1.24E-05	8.33E-06	1.15E-12	8.39E-03
F20	Best	-3.32E+00	-3.32E+00	-3.32E+00	-3.32E+00	-3.32E+00	-3.19E+00	-3.32E+00	-3.17E+00	-3.32E+00	-3.25E+00
	Median	-3.32E+00	-3.20E+00	-3.26E+00	-3.32E+00	-3.20E+00	-3.08E+00	-3.17E+00	-3.01E+00	-3.20E+00	-2.91E+00
	Mean	-3.27E+00	-3.26E+00	-3.25E+00	-3.25E+00	-3.22E+00	-3.05E+00	-3.15E+00	-2.93E+00	-3.24E+00	-2.81E+00
	Worst	-3.20E+00	-3.20E+00	-3.09E+00	-3.05E+00	-3.18E+00	-2.83E+00	-2.88E+00	-1.45E+00	-3.20E+00	-1.17E+00
	STD	3.52E-03	3.53E-03	6.26E-03	8.35E-03	2.30E-03	8.34E-03	1.13E-02	1.20E-01	3.03E-03	1.50E-01
F21	Best	-1.02E+01	-1.02E+01	-1.02E+01	-1.02E+01	-1.02E+01	-6.78E+00	-5.06E+00	-4.88E+00	-1.02E+01	-1.02E+01
	Median	-1.02E+01	-5.06E+00	-1.02E+01	-1.01E+01	-1.02E+01	-3.60E+00	-5.05E+00	-8.81E-01	-7.63E+00	-1.02E+01
	Mean	-9.98E+00	-6.44E+00	-9.31E+00	-8.37E+00	-7.23E+00	-3.75E+00	-5.05E+00	-2.08E+00	-7.54E+00	-1.02E+01
	Worst	-5.06E+00	-2.63E+00	-5.06E+00	-2.63E+00	-2.63E+00	-2.14E+00	-5.05E+00	-4.97E-01	-2.63E+00	-1.01E+01
	STD	8.37E-01	7.50E+00	3.57E+00	6.50E+00	1.05E+01	1.30E+00	4.88E-06	2.96E+00	7.03E+00	5.09E-06
F22	Best	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-7.24E+00	-1.03E+01	-8.29E+00	-1.04E+01	-1.04E+01

	Median	-1.04E+01	-5.11E+00	-1.04E+01	-7.73E+00	-1.04E+01	-4.04E+00	-5.09E+00	-3.24E+00	-1.04E+01	-1.04E+01
	Mean	-9.38E+00	-6.35E+00	-1.02E+01	-7.57E+00	-8.12E+00	-4.43E+00	-5.26E+00	-3.29E+00	-8.41E+00	-1.04E+01
	Worst	-2.77E+00	-2.77E+00	-5.13E+00	-2.77E+00	-2.75E+00	-1.85E+00	-5.08E+00	-9.06E-01	-2.77E+00	-1.04E+01
	STD	6.74E+00	1.02E+01	8.96E-01	8.22E+00	1.08E+01	1.93E+00	8.81E-01	3.83E+00	8.36E+00	9.33E-06
F23	Best	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01	-7.27E+00	-1.05E+01	-7.86E+00	-1.05E+01	-1.05E+01
	Median	-1.05E+01	-1.05E+01	-1.05E+01	-7.45E+00	-1.05E+01	-3.67E+00	-5.13E+00	-4.35E+00	-1.05E+01	-1.05E+01
	Mean	-1.03E+01	-7.32E+00	-9.99E+00	-7.23E+00	-8.16E+00	-3.67E+00	-5.47E+00	-4.13E+00	-9.39E+00	-1.05E+01
	Worst	-3.84E+00	-2.43E+00	-2.42E+00	-2.42E+00	-2.42E+00	-1.77E+00	-5.11E+00	-9.41E-01	-2.43E+00	-1.05E+01
	STD	1.45E+00	1.23E+01	4.10E+00	1.11E+01	1.16E+01	1.25E+00	1.67E+00	3.51E+00	6.80E+00	5.85E-05

Tables 5 through 7 offer a comprehensive comparison of the search results achieved by ECO and nine popular optimization algorithms across F1 through F23, employing five evaluation metrics. The average convergence curves of the ten algorithms are depicted in Fig. 7.

The results reveal that ECO outperforms the other algorithms across most of the tested functions and consistently ranks highly across various test functions. Moreover, ECO demonstrates superior convergence abilities in the majority of the test functions. Therefore, ECO emerges as a comprehensive optimization algorithm with robust search capabilities.

Notably, in functions F9 through F12, F16, F17, F18, F19, and F20, ECO showcases exceptional search capability and rapid convergence, enabling it to swiftly identify optimal or near-optimal solutions.

Table 8 Rank of classical benchmark functions

	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
F1	3	8	4	2	7	5	1	10	9	6
F2	4	10	5	2	8	1	3	7	9	6
F3	2	8	4	10	7	5	1	9	6	3
F4	2	8	4	10	7	5	1	9	6	3
F5	4	8	3	5	7	6	1	10	9	2
F6	4	3	7	6	1	9	2	10	8	5
F7	3	10	5	6	9	1	2	8	7	4
F8	3	8	7	4	6	9	2	10	5	1
F9	1	9	6	1	8	1	1	7	10	5
F10	3	9	5	4	8	1	1	10	7	6
F11	3	9	5	4	8	6	1	10	7	2
F12	1	7	4	5	6	8	1	10	9	3
F13	3	9	6	5	8	7	1	10	4	2
F14	2	7	9	8	5	10	4	6	1	3
F15	5	6	7	3	8	10	1	4	9	2
F16	1	1	1	1	1	1	1	1	1	1
F17	1	1	1	1	1	10	1	9	1	8
F18	1	1	6	6	1	10	1	8	1	9
F19	1	1	5	7	1	9	6	8	1	10
F20	1	2	4	3	6	8	7	9	5	10
F21	2	7	3	4	6	9	8	10	5	1
F22	3	7	2	6	5	9	8	10	4	1
F23	2	6	3	7	5	10	8	9	4	1
Average Rank	2.39	6.30	4.61	4.78	5.61	6.52	2.74	8.43	5.57	4.09
Final Ranking	1	8	4	5	6	9	2	10	7	3

The performance evaluation of the ECO algorithm and nine existing frontier algorithms on F1 through F23 is presented in Table 8. It's worth noting that when ranking these ten algorithms, the criteria are prioritized in the order of mean and variance.

The results showcase that the ECO algorithm outperforms the other nine optimization algorithms, securing a significant lead with an impressive average ranking of 2.39. This remarkable performance reaffirms the superiority of the ECO algorithm in solving the optimization problems under consideration.

This thorough evaluation highlights the effectiveness and competitiveness of the ECO algorithm when compared to other peers. It confirms the robust performance of ECO across diverse function types and underscores its suitability for tackling complex optimization problems.

3.3 Comparison of different algorithms on CEC2021 test functions

To deepen our assessment of the effectiveness of the proposed ECO algorithm and scrutinize its ability to explore, exploit, and avoid local optima, we subjected it to one of the most rigorous benchmarks available: the CEC2021 test function suite. We compared the performance of ECO with the nine well-known optimization algorithms mentioned above. All algorithms underwent independent runs 30 times with 500 iterations and a population size of 40.

Tables 9 through 14 document the search results as well as the ranking of the ten search algorithms for dimensions (dim) 2, 10, and 20, respectively. It's important to note that when ranking these ten algorithms, the criteria are prioritized in the order of mean and variance.

Table 9 Results of CEC2021 benchmark functions (dim=2)

	Item	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
	Best	0.00E+00	5.07E-09	2.44E-289	2.94E-150	3.18E-02	0.00E+00	3.45E-128	2.58E-88	1.39E-04	3.02E-32
	Median	0.00E+00	7.58E-06	5.87E-253	4.29E-137	3.76E+01	0.00E+00	3.09E-113	6.95E-73	4.86E-02	4.97E-15
F1	Mean	0.00E+00	2.69E+02	2.57E-236	1.20E-121	2.91E+02	0.00E+00	1.86E-104	6.15E-70	9.63E-02	7.56E-11
	Worst	0.00E+00	5.55E+03	7.48E-235	3.60E-120	2.11E+03	0.00E+00	5.57E-103	1.48E-68	5.52E-01	1.87E-09
	STD	0.00E+00	1.02E+06	0.00E+00	4.18E-241	2.73E+05	0.00E+00	9.99E-207	7.13E-138	1.43E-02	1.13E-19
	Best	0.00E+00	4.55E-12	0.00E+00	0.00E+00	1.93E-12	0.00E+00	0.00E+00	0.00E+00	4.25E-07	0.00E+00
	Median	0.00E+00	3.12E-01	0.00E+00	0.00E+00	3.12E-01	0.00E+00	0.00E+00	0.00E+00	3.44E-04	0.00E+00
F2	Mean	0.00E+00	3.64E+00	8.33E-02	0.00E+00	7.67E-01	0.00E+00	0.00E+00	0.00E+00	1.07E+01	6.94E-12
	Worst	0.00E+00	1.71E+01	6.24E-01	0.00E+00	1.71E+01	0.00E+00	0.00E+00	0.00E+00	1.18E+02	1.58E-10
	STD	0.00E+00	4.44E+01	4.51E-02	0.00E+00	9.21E+00	0.00E+00	0.00E+00	0.00E+00	8.68E+02	8.49E-22
	Best	0.00E+00	8.32E-13	0.00E+00	0.00E+00	4.49E-13	0.00E+00	0.00E+00	0.00E+00	1.33E-05	0.00E+00
	Median	0.00E+00	2.04E+00	2.04E+00	0.00E+00	2.28E-11	0.00E+00	0.00E+00	0.00E+00	2.04E+00	1.20E-23
F3	Mean	0.00E+00	1.18E+00	1.42E+00	6.57E-33	8.66E-01	0.00E+00	0.00E+00	1.93E-01	1.56E+00	3.69E-14
	Worst	0.00E+00	2.34E+00	2.13E+00	1.97E-31	2.34E+00	0.00E+00	0.00E+00	2.50E+00	2.04E+00	5.49E-13
	STD	0.00E+00	1.07E+00	8.76E-01	1.25E-63	1.13E+00	0.00E+00	0.00E+00	3.89E-01	7.43E-01	1.87E-26
	Best	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.13E-14	0.00E+00
	Median	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.70E-10	0.00E+00
F4	Mean	0.00E+00	6.58E-03	2.30E-03	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.32E-03	0.00E+00
	Worst	0.00E+00	4.93E-02	1.97E-02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.98E-02	0.00E+00
	STD	0.00E+00	1.84E-04	3.69E-05	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.44E-05	0.00E+00
	Best	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	Median	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
F5	Mean	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	Worst	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	STD	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	Best	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.53E-10	0.00E+00
	Median	0.00E+00	6.69E-15	0.00E+00	0.00E+00	2.22E-16	0.00E+00	0.00E+00	0.00E+00	8.92E-09	0.00E+00
F6	Mean	0.00E+00	3.89E-14	0.00E+00	0.00E+00	6.03E-16	0.00E+00	0.00E+00	0.00E+00	4.97E-08	4.11E-15
	Worst	0.00E+00	2.23E-13	0.00E+00	0.00E+00	2.78E-15	0.00E+00	0.00E+00	0.00E+00	5.37E-07	1.23E-13
	STD	0.00E+00	4.40E-27	0.00E+00	0.00E+00	5.01E-31	0.00E+00	0.00E+00	0.00E+00	1.25E-14	4.84E-28
	Best	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	Median	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
F7	Mean	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	Worst	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	STD	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
	Best	0.00E+00	1.41E-11	0.00E+00	0.00E+00	2.37E-12	0.00E+00	0.00E+00	0.00E+00	3.76E-05	0.00E+00
	Median	0.00E+00	2.71E-10	0.00E+00	0.00E+00	3.95E-11	0.00E+00	0.00E+00	0.00E+00	6.92E-04	7.40E-15
F8	Mean	0.00E+00	7.50E-01	0.00E+00	1.23E-17	7.17E-11	0.00E+00	0.00E+00	0.00E+00	7.93E-04	3.00E-11
	Worst	0.00E+00	1.12E+01	0.00E+00	3.70E-16	5.59E-10	0.00E+00	0.00E+00	0.00E+00	2.39E-03	8.98E-10
	STD	0.00E+00	7.87E+00	0.00E+00	4.41E-33	1.13E-20	0.00E+00	0.00E+00	0.00E+00	3.19E-07	2.60E-20
	Best	0.00E+00	2.12E-06	1.01E-286	5.07E-154	3.15E-07	0.00E+00	4.17E-139	1.37E-86	4.00E-03	8.88E-15
	Median	0.00E+00	1.27E-05	2.43E-255	6.46E-132	7.70E-06	0.00E+00	1.90E-119	4.66E-80	2.86E-02	5.40E-10
F9	Mean	0.00E+00	1.40E-05	1.55E-236	3.26E-15	7.29E-06	0.00E+00	1.17E-115	1.75E-75	2.89E-02	8.25E-06
	Worst	0.00E+00	2.98E-05	4.65E-235	8.88E-15	1.44E-05	0.00E+00	2.84E-114	3.83E-74	6.08E-02	2.43E-04
	STD	0.00E+00	4.74E-11	0.00E+00	1.83E-29	1.79E-11	0.00E+00	2.68E-229	4.98E-149	2.18E-04	1.90E-09
	Best	0.00E+00	1.33E-03	1.26E-04	2.44E-04	7.62E-04	0.00E+00	5.66E-128	4.55E-05	1.99E-02	3.09E-40
	Median	0.00E+00	2.74E-03	4.80E-04	3.61E-03	2.20E-03	0.00E+00	2.34E-07	4.52E-04	5.50E-02	3.30E-04
F10	Mean	0.00E+00	3.07E-03	5.74E-04	5.36E-03	2.22E-03	0.00E+00	4.78E-05	6.89E-04	5.48E-02	5.31E-04
	Worst	0.00E+00	5.90E-03	1.35E-03	1.93E-02	4.63E-03	0.00E+00	3.49E-04	2.92E-03	1.09E-01	2.11E-03
	STD	0.00E+00	1.61E-06	1.12E-07	2.72E-05	6.88E-07	0.00E+00	8.74E-09	4.11E-07	2.85E-04	3.18E-07

Table 10 Rank of CEC2021 benchmark functions (dim=2)

	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
F1	1	9	3	4	10	1	5	6	8	7
F2	1	9	7	1	8	1	1	1	10	6
F3	1	8	9	4	7	1	1	6	10	5
F4	1	10	9	1	1	1	1	1	8	1
F5	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
F6	1	9	3	4	10	1	5	6	8	7
F7	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
F8	1	10	1	6	8	1	1	1	9	7
F9	1	9	3	6	7	1	4	5	10	8
F10	1	8	5	9	7	1	3	6	10	4
Average Rank	1	9	5	4.375	7.25	1	2.625	4	9.125	5.625
Final Ranking	1	9	6	5	8	1	3	4	10	7

Table 11 Results of CEC2021 benchmark functions (dim=10)

	Item	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
	Best	0.00E+00	2.46E-03	1.21E-62	6.14E-87	4.49E+00	0.00E+00	4.40E-114	2.13E-12	4.12E+03	1.69E-20
	Median	0.00E+00	9.09E+02	5.87E-60	2.48E-82	5.64E+02	0.00E+00	1.59E-101	1.14E-09	1.25E+04	2.79E-16
F1	Mean	0.00E+00	1.70E+03	1.08E-57	4.87E-78	1.49E+03	0.00E+00	3.25E-92	2.16E-08	1.37E+04	1.68E-09
	Worst	0.00E+00	9.83E+03	2.08E-56	3.97E-77	5.97E+03	0.00E+00	9.58E-91	4.74E-07	3.17E+04	2.99E-08
	STD	0.00E+00	5.31E+06	1.42E-113	1.27E-154	2.85E+06	0.00E+00	2.95E-182	7.28E-15	4.76E+07	3.69E-17
F2	Best	0.00E+00	4.45E+02	0.00E+00	0.00E+00	7.02E+00	0.00E+00	0.00E+00	0.00E+00	3.94E+00	0.00E+00

	Median	0.00E+00	9.38E+02	7.30E-02	0.00E+00	5.34E+02	0.00E+00	0.00E+00	3.96E-09	5.63E+02	0.00E+00
	Mean	0.00E+00	1.01E+03	5.58E+00	1.08E+02	5.70E+02	0.00E+00	0.00E+00	4.37E+01	5.17E+02	1.50E-11
	Worst	0.00E+00	1.69E+03	1.03E+02	1.48E+03	1.40E+03	0.00E+00	0.00E+00	5.51E+02	9.81E+02	2.82E-10
	STD	0.00E+00	1.04E+05	3.46E+02	1.17E+05	1.08E+05	0.00E+00	0.00E+00	1.72E+04	5.10E+04	2.88E-21
F3	Best	0.00E+00	1.69E+01	0.00E+00	0.00E+00	9.95E+00	0.00E+00	0.00E+00	3.57E-14	9.15E+00	1.77E-30
	Median	0.00E+00	3.42E+01	3.53E+01	0.00E+00	3.08E+01	0.00E+00	0.00E+00	2.35E-07	2.60E+01	3.18E-22
	Mean	0.00E+00	3.79E+01	3.35E+01	6.57E-33	3.25E+01	0.00E+00	0.00E+00	1.04E+01	2.60E+01	3.57E-15
	Worst	0.00E+00	9.55E+01	5.93E+01	1.97E-31	7.86E+01	0.00E+00	0.00E+00	9.68E+01	4.18E+01	8.92E-14
	STD	0.00E+00	2.82E+02	3.09E+02	1.25E-63	2.81E+02	0.00E+00	0.00E+00	6.85E+02	7.23E+01	2.63E-28
F4	Best	0.00E+00	4.14E-01	0.00E+00	0.00E+00	3.46E-01	0.00E+00	0.00E+00	0.00E+00	6.08E-01	0.00E+00
	Median	0.00E+00	1.30E+00	2.15E-01	0.00E+00	1.27E+00	0.00E+00	0.00E+00	1.00E-08	1.39E+00	0.00E+00
	Mean	0.00E+00	1.47E+00	4.66E-01	1.22E-01	1.54E+00	0.00E+00	0.00E+00	1.01E+00	1.43E+00	0.00E+00
	Worst	0.00E+00	4.40E+00	1.87E+00	1.59E+00	3.37E+00	0.00E+00	0.00E+00	6.60E+00	2.42E+00	0.00E+00
	STD	0.00E+00	5.98E-01	3.21E-01	1.15E-01	7.11E-01	0.00E+00	0.00E+00	3.07E+00	2.68E-01	0.00E+00
F5	Best	0.00E+00	8.79E+02	4.35E-25	8.14E-81	7.17E+02	0.00E+00	3.75E-110	2.09E-16	2.59E+02	1.79E-18
	Median	0.00E+00	7.68E+03	1.27E-22	5.28E-23	6.49E+03	0.00E+00	2.15E-89	4.40E-12	8.79E+02	2.33E-13
	Mean	0.00E+00	1.65E+04	2.91E-01	1.42E-16	8.29E+03	0.00E+00	3.02E-74	2.22E-01	9.10E+02	1.57E-10
	Worst	0.00E+00	8.99E+04	2.64E+00	2.82E-15	2.95E+04	0.00E+00	9.07E-73	6.49E+00	1.92E+03	4.41E-09
	STD	0.00E+00	4.05E+08	5.21E-01	2.80E-31	5.68E+07	0.00E+00	2.65E-146	1.36E+00	8.47E+04	6.26E-19
F6	Best	0.00E+00	5.72E+00	2.03E-02	1.36E-09	2.56E+00	0.00E+00	0.00E+00	7.89E-04	2.32E+00	2.98E-08
	Median	0.00E+00	2.52E+01	1.20E-01	7.31E-02	1.58E+01	0.00E+00	1.49E-12	1.05E-01	1.61E+01	2.67E-05
	Mean	0.00E+00	3.94E+01	1.03E+00	1.63E+01	2.98E+01	0.00E+00	8.49E-06	2.84E+00	3.76E+01	4.95E-05
	Worst	0.00E+00	1.59E+02	1.79E+01	2.93E+02	2.43E+02	0.00E+00	1.39E-04	6.62E+01	1.45E+02	2.35E-04
	STD	0.00E+00	1.77E+03	1.02E+01	3.73E+03	2.17E+03	0.00E+00	6.59E-10	1.43E+02	2.37E+03	3.91E-09
F7	Best	0.00E+00	8.34E+02	5.67E-03	5.24E-04	6.73E+02	0.00E+00	8.91E-114	6.78E-04	2.72E+01	4.22E-08
	Median	0.00E+00	3.19E+03	3.34E-02	1.19E-02	2.85E+03	0.00E+00	1.93E-13	3.05E-02	2.55E+02	4.02E-06
	Mean	0.00E+00	4.01E+03	2.92E-01	4.22E-02	2.96E+03	0.00E+00	7.78E-07	1.52E-01	2.48E+02	3.70E-05
	Worst	0.00E+00	1.18E+04	2.56E+00	2.84E-01	1.02E+04	0.00E+00	1.60E-05	2.76E+00	5.23E+02	4.34E-04
	STD	0.00E+00	8.85E+06	3.63E-01	4.12E-03	4.34E+06	0.00E+00	8.80E-12	2.42E-01	2.13E+04	6.82E-09
F8	Best	0.00E+00	4.33E+01	0.00E+00	0.00E+00	4.21E+01	0.00E+00	0.00E+00	1.48E-14	2.17E+01	0.00E+00
	Median	0.00E+00	4.12E+02	0.00E+00	0.00E+00	1.97E+02	0.00E+00	0.00E+00	4.42E-12	9.09E+01	5.55E-16
	Mean	0.00E+00	4.24E+02	0.00E+00	0.00E+00	2.35E+02	0.00E+00	0.00E+00	2.37E-08	2.31E+02	2.04E-11
	Worst	0.00E+00	9.62E+02	0.00E+00	0.00E+00	6.34E+02	0.00E+00	0.00E+00	5.22E-07	1.07E+03	5.28E-10
	STD	0.00E+00	6.05E+04	0.00E+00	0.00E+00	2.24E+04	0.00E+00	0.00E+00	9.11E-15	7.87E+04	8.99E-21
F9	Best	0.00E+00	1.41E-04	8.88E-15	2.42E-95	5.36E-05	0.00E+00	1.67E-122	2.96E-10	3.17E-01	1.78E-14
	Median	0.00E+00	4.67E+00	8.88E-15	8.88E-15	8.77E-05	0.00E+00	1.41E-111	5.23E-08	7.13E-01	1.70E-09
	Mean	0.00E+00	1.25E+01	1.15E-14	9.47E-15	1.48E+00	0.00E+00	2.98E-103	3.26E-07	1.73E+00	1.24E-06
	Worst	0.00E+00	6.79E+01	1.78E-14	1.78E-14	6.95E+00	0.00E+00	8.37E-102	4.13E-06	6.08E+00	1.78E-05
	STD	0.00E+00	5.08E+02	1.66E-29	2.59E-29	6.44E+00	0.00E+00	2.26E-204	6.00E-13	3.13E+00	1.39E-11
F10	Best	0.00E+00	4.82E+01	4.03E-03	1.76E-02	4.85E+01	0.00E+00	6.58E-114	4.73E-03	4.84E+01	1.58E-04
	Median	0.00E+00	4.97E+01	4.96E+01	5.59E-02	5.07E+01	0.00E+00	1.52E-06	6.10E+01	4.91E+01	9.45E-04
	Mean	0.00E+00	5.24E+01	5.10E+01	6.33E-02	5.91E+01	0.00E+00	2.28E-04	4.65E+01	5.21E+01	1.29E-03
	Worst	0.00E+00	8.32E+01	7.82E+01	1.42E-01	1.09E+02	0.00E+00	3.55E-03	8.54E+01	8.00E+01	3.52E-03
	STD	0.00E+00	7.02E+01	1.56E+02	1.15E-03	2.46E+02	0.00E+00	5.55E-07	1.14E+03	8.51E+01	9.73E-07

Table 12 Rank of CEC2021 benchmark functions (dim=10)

	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
F1	1	9	5	4	8	1	3	7	10	6
F2	**1**	**10**	**5**	**7**	**9**	**1**	**1**	**6**	**8**	**4**
F3	1	10	9	4	8	1	1	6	7	5
F4	**1**	**9**	**6**	**5**	**10**	**1**	**1**	**7**	**8**	**1**
F5	1	10	7	4	9	1	3	6	8	5
F6	**1**	**9**	**5**	**4**	**8**	**1**	**3**	**7**	**10**	**6**
F7	1	10	7	5	9	1	3	6	8	4
F8	**1**	**10**	**1**	**1**	**9**	**1**	**1**	**7**	**8**	**6**
F9	1	10	5	4	8	1	3	6	9	7
F10	**1**	**9**	**7**	**5**	**10**	**1**	**3**	**6**	**8**	**4**
Average Rank	1	9.6	5.7	4.3	8.8	1	2.2	6.4	8.4	4.8
Final Ranking	1	10	6	4	9	1	3	7	8	5

Table 13 Results of CEC2021 benchmark functions (dim=20)

	Item	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
F1	Best	0.00E+00	5.94E+00	2.99E-37	1.94E-86	2.09E-01	0.00E+00	5.75E-108	3.51E-01	7.58E+04	1.03E-27

	Median	0.00E+00	1.04E+03	6.71E-35	1.78E-79	3.46E+02	4.93E-260	1.11E-101	1.50E+02	1.37E+05	3.43E-14
	Mean	0.00E+00	1.44E+03	2.40E-34	6.79E-73	8.26E+02	1.76E-83	1.54E-94	2.91E+03	1.51E+05	1.62E-07
	Worst	0.00E+00	4.87E+03	2.57E-33	2.03E-71	4.80E+03	5.27E-82	4.58E-93	7.25E+04	2.62E+05	4.84E-06
	STD	0.00E+00	1.84E+06	2.86E-67	1.33E-143	1.22E+06	8.96E-165	6.77E-187	1.68E+08	2.27E+09	7.56E-13
F2	Best	0.00E+00	1.37E+03	0.00E+00	0.00E+00	2.18E+02	0.00E+00	0.00E+00	4.41E-02	1.34E+03	0.00E+00
	Median	0.00E+00	2.36E+03	5.46E+00	0.00E+00	1.73E+03	0.00E+00	0.00E+00	1.25E+02	1.86E+03	0.00E+00
	Mean	0.00E+00	2.29E+03	9.10E+00	1.60E+02	1.72E+03	0.00E+00	0.00E+00	4.78E+02	2.05E+03	3.50E-11
	Worst	0.00E+00	3.46E+03	3.04E+01	2.30E+03	2.95E+03	0.00E+00	0.00E+00	2.87E+03	3.07E+03	7.77E-10
	STD	0.00E+00	3.67E+05	1.04E+02	2.55E+05	4.24E+05	0.00E+00	0.00E+00	5.51E+05	2.39E+05	2.11E-20
F3	Best	0.00E+00	6.19E+01	1.77E-30	0.00E+00	4.78E+01	0.00E+00	0.00E+00	1.04E-03	3.31E+01	0.00E+00
	Median	0.00E+00	1.36E+02	7.01E+01	0.00E+00	9.55E+01	0.00E+00	0.00E+00	1.61E+01	7.56E+01	1.39E-21
	Mean	0.00E+00	1.41E+02	7.30E+01	6.57E-33	9.79E+01	6.57E-33	0.00E+00	3.01E+01	7.80E+01	2.39E-15
	Worst	0.00E+00	2.24E+02	1.82E+02	1.97E-31	1.70E+02	1.97E-31	0.00E+00	1.49E+02	1.29E+02	7.15E-14
	STD	0.00E+00	1.70E+03	3.47E+03	1.25E-63	9.66E+02	1.25E-63	0.00E+00	1.37E+03	4.32E+02	1.65E-28
F4	Best	0.00E+00	1.66E+00	0.00E+00	0.00E+00	2.15E+00	0.00E+00	0.00E+00	7.93E-08	3.61E+00	0.00E+00
	Median	0.00E+00	6.93E+00	4.99E-02	0.00E+00	5.86E+00	0.00E+00	0.00E+00	6.60E+00	6.44E+00	0.00E+00
	Mean	0.00E+00	6.25E+00	3.87E-01	2.18E-01	6.07E+00	0.00E+00	0.00E+00	6.22E+00	6.42E+00	0.00E+00
	Worst	0.00E+00	9.44E+00	3.57E+00	3.55E+00	1.09E+01	0.00E+00	0.00E+00	1.42E+01	1.04E+01	0.00E+00
	STD	0.00E+00	4.21E+00	5.31E-01	5.12E-01	4.54E+00	0.00E+00	0.00E+00	2.00E+01	2.75E+00	0.00E+00
F5	Best	0.00E+00	3.97E+04	1.30E-21	1.05E-83	6.72E+03	0.00E+00	2.64E-109	2.35E-05	5.14E+03	1.54E-19
	Median	0.00E+00	1.64E+05	7.24E-14	2.70E-25	1.20E+05	0.00E+00	6.40E-94	2.25E-01	1.86E+04	1.29E-13
	Mean	0.00E+00	2.47E+05	2.96E+00	4.65E-16	2.35E+05	0.00E+00	1.22E-76	1.16E+01	2.57E+04	7.63E-09
	Worst	0.00E+00	7.62E+05	1.94E+01	1.39E-14	1.13E+06	0.00E+00	3.67E-75	1.14E+02	7.38E+04	2.29E-07
	STD	0.00E+00	3.75E+10	2.23E+01	6.27E-30	7.97E+10	0.00E+00	4.33E-151	6.44E+02	3.47E+08	1.69E-15
F6	Best	-2.22E-16	6.60E+01	2.65E-02	-2.22E-16	1.09E+01	-1.11E-16	-1.11E-16	2.55E-01	6.40E+01	3.44E-06
	Median	-1.11E-16	5.35E+02	1.23E+00	1.41E-01	3.19E+02	0.00E+00	1.71E-08	2.10E+00	2.23E+02	6.49E-05
	Mean	-1.11E-16	5.42E+02	2.62E+00	7.31E+01	3.27E+02	-7.40E-18	2.83E-05	9.27E+00	2.51E+02	2.54E-04
	Worst	0.00E+00	1.17E+03	2.48E+01	7.20E+02	7.91E+02	0.00E+00	3.09E-04	1.61E+02	5.38E+02	1.85E-03
	STD	1.23E-32	8.95E+04	2.34E+01	3.11E+04	4.52E+04	7.67E-34	4.84E-09	8.25E+02	1.71E+04	2.17E-07
F7	Best	-2.22E-16	6.81E+03	3.45E-02	5.38E-03	7.28E+03	0.00E+00	-2.22E-16	1.34E-01	1.72E+03	1.19E-07
	Median	0.00E+00	7.44E+04	3.19E-01	6.79E-02	4.51E+04	0.00E+00	3.28E-14	5.80E-01	5.01E+03	1.77E-05
	Mean	-5.92E-17	9.24E+04	2.13E+00	6.96E-02	6.19E+04	1.60E-270	5.78E-07	2.32E+00	5.75E+03	3.25E-05
	Worst	0.00E+00	2.40E+05	2.11E+01	1.92E-01	2.84E+05	4.80E-269	1.15E-05	2.72E+01	2.13E+04	2.32E-04
	STD	9.64E-33	3.98E+09	2.03E+01	2.42E-03	3.85E+09	0.00E+00	5.04E-12	2.90E+01	1.34E+07	2.28E-09
F8	Best	0.00E+00	5.78E+02	0.00E+00	0.00E+00	3.66E+02	0.00E+00	0.00E+00	3.37E-05	4.05E+02	0.00E+00
	Median	0.00E+00	1.47E+03	0.00E+00	0.00E+00	7.36E+02	0.00E+00	0.00E+00	3.95E-03	1.75E+03	7.10E-14
	Mean	0.00E+00	1.44E+03	0.00E+00	0.00E+00	8.20E+02	0.00E+00	0.00E+00	1.86E+00	1.64E+03	5.98E-11
	Worst	0.00E+00	2.67E+03	0.00E+00	0.00E+00	1.45E+03	0.00E+00	0.00E+00	3.63E+01	3.12E+03	1.02E-09
	STD	0.00E+00	2.34E+05	0.00E+00	0.00E+00	8.88E+04	0.00E+00	0.00E+00	4.77E+01	5.33E+05	3.74E-20
F9	Best	0.00E+00	3.15E+00	1.78E-14	8.96E-96	1.32E-04	0.00E+00	1.34E-115	1.76E-03	2.35E+00	7.11E-14
	Median	0.00E+00	5.82E+01	3.55E-14	8.88E-15	1.79E-04	5.96E-250	1.75E-110	2.06E-02	5.79E+00	2.74E-09
	Mean	0.00E+00	4.94E+01	3.29E-14	9.18E-15	1.19E+01	6.50E-83	1.49E-99	3.49E-02	5.52E+00	1.37E-07
	Worst	0.00E+00	1.14E+02	3.55E-14	1.78E-14	9.33E+01	1.95E-81	3.07E-98	2.57E-01	8.62E+00	1.74E-06
	STD	0.00E+00	1.42E+03	3.76E-29	3.41E-29	6.12E+02	1.23E-163	3.57E-197	2.65E-03	3.73E+00	1.23E-13
F10	Best	0.00E+00	4.98E+01	5.19E+01	3.64E-02	4.97E+01	0.00E+00	4.21E-120	7.79E-02	4.99E+01	2.68E-04
	Median	0.00E+00	5.13E+01	8.13E+01	9.98E-02	7.01E+01	3.12E-252	1.66E-06	9.72E+01	5.08E+01	9.50E-04
	Mean	0.00E+00	5.23E+01	7.55E+01	9.69E-02	7.68E+01	1.54E-60	1.02E-04	8.40E+01	5.35E+01	1.39E-03
	Worst	0.00E+00	8.05E+01	9.09E+01	1.83E-01	1.80E+02	4.62E-59	1.73E-03	1.33E+02	7.91E+01	5.04E-03
	STD	0.00E+00	2.84E+01	1.69E+02	1.90E-03	9.18E+02	6.88E-119	9.79E-08	1.54E+03	6.71E+01	1.48E-06

Table 14 Rank of CEC2021 benchmark functions (dim=20)

	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
F1	1	8	5	4	7	3	2	9	10	6
F2	1	10	5	6	8	1	1	7	9	4
F3	1	10	7	3	9	3	1	6	8	5

F4	1	9	6	5	7	1	1	8	10	1
F5	1	10	6	4	9	1	3	7	8	5
F6	1	8	5	4	7	3	2	9	10	6
F7	1	10	6	5	9	2	3	7	8	4
F8	1	9	1	1	8	1	1	7	10	6
F9	1	10	5	4	9	3	2	7	8	6
F10	1	6	8	5	9	2	3	10	7	4
Average Rank	1	9	5.4	4.1	8.2	2	1.9	7.7	8.8	4.7
Final Ranking	1	10	6	4	8	3	2	7	9	5

The results show that ECO outperforms the nine frontier algorithms being compared. More noteworthy is that ECO obtained the optimal solution for all ten test functions within 500 iterations whether in 2, 10, or 20 dimensions, which indicates that ECO is extremely powerful in searching and can deal with high local optimality of combinatorial functions. There is no doubt that ECO is the winner in the CEC2021.

The average convergence curves of the ten algorithms are illustrated in Fig. 8 to Fig. 10 for the ten tested function species in 2, 10, and 20 dimensions, respectively.

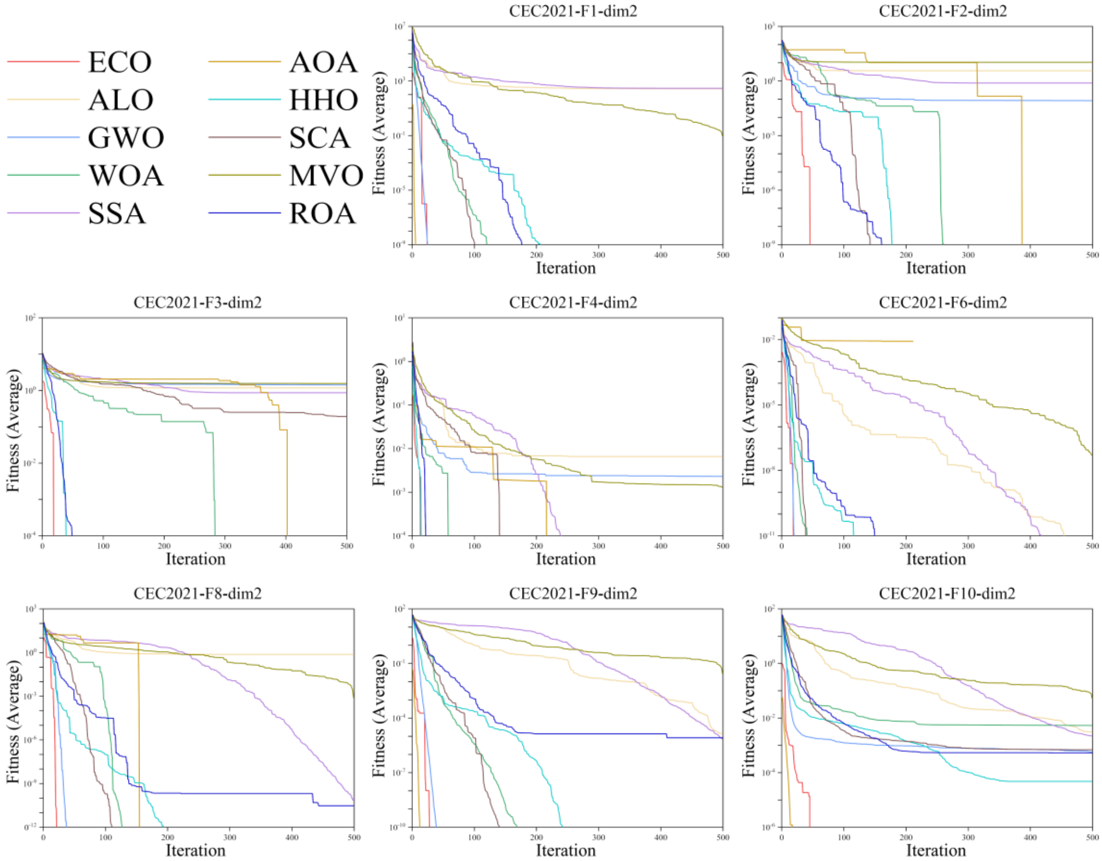
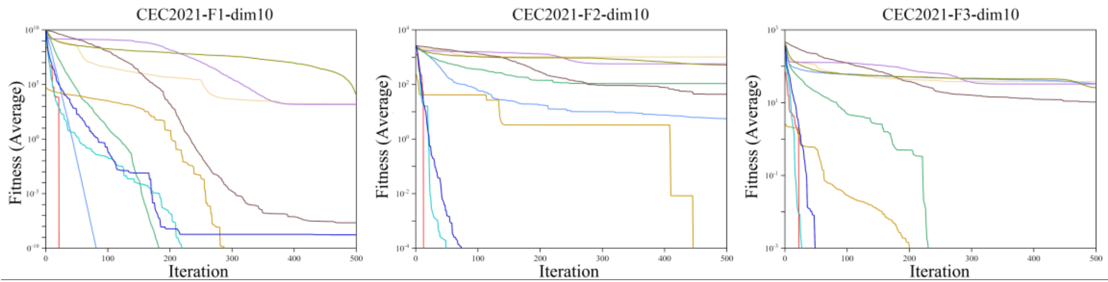


Fig. 8 Comparison of CEC2021 benchmark functions (dim=2)



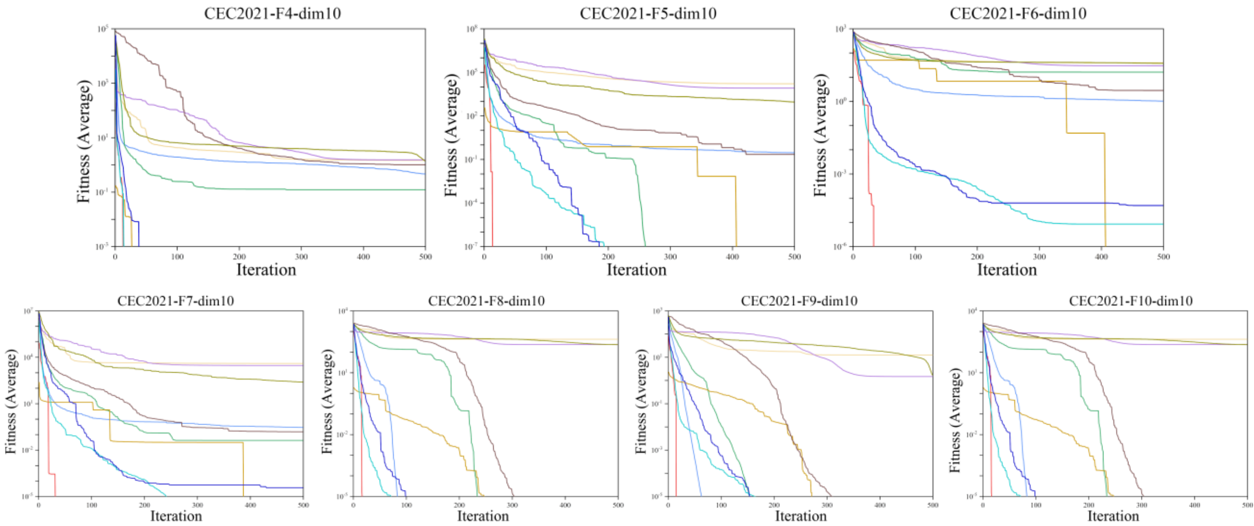


Fig. 9 Comparison of CEC2021 benchmark functions (dim=10)

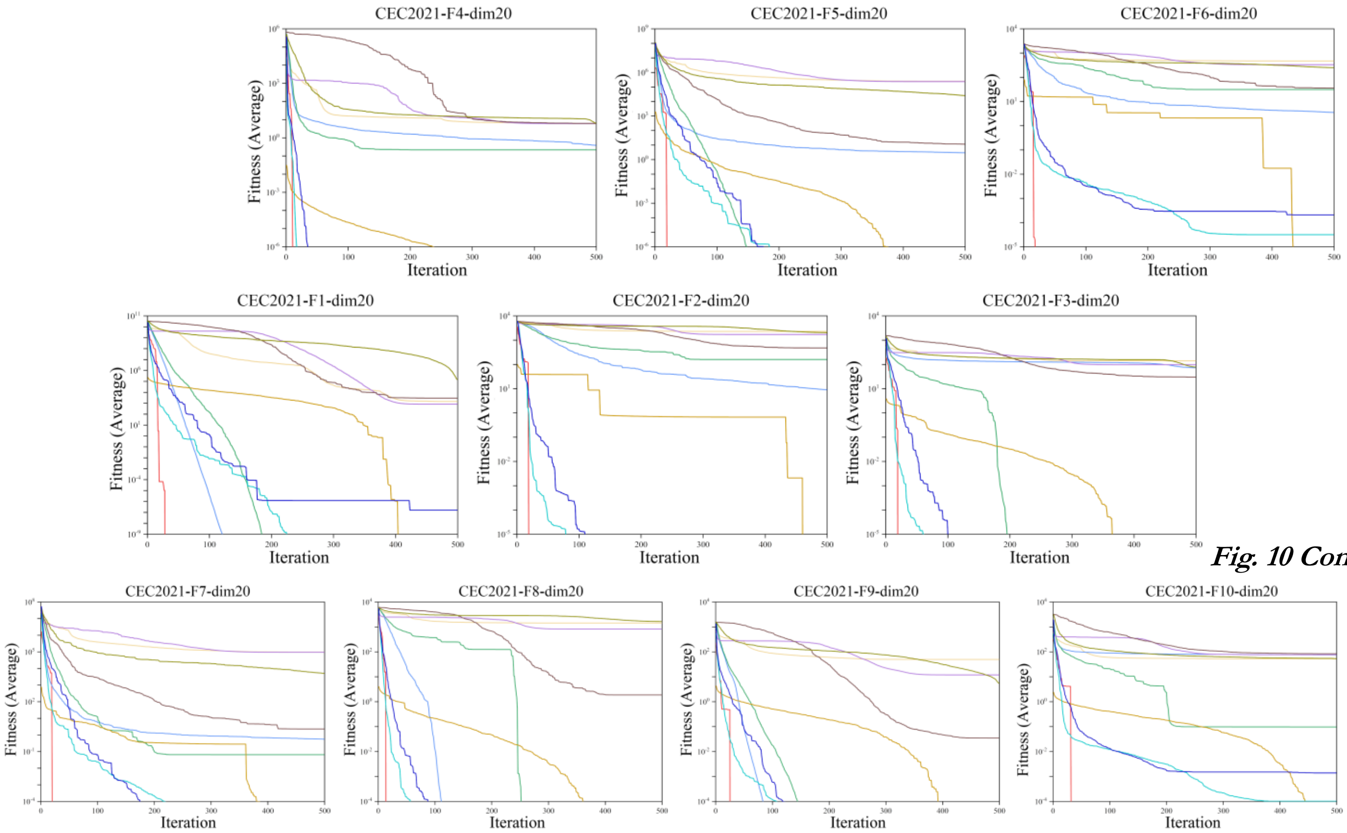


Fig. 10 Comparison of CEC2021 benchmark functions (dim=20)

3.4 Real-world applications

In this section, we leverage the recently introduced ECO algorithm to address six distinct engineering design problems [59] and present the outcomes. These engineering optimization problems have been formulated to seek optimal solutions while adhering to specific conditions and constraints.

Conventionally, metaheuristic algorithms aren't inherently designed to directly address constrained optimization problems. Nonetheless, through the integration of constraint handling techniques (CHTs), these algorithms can adeptly handle both the objective function and associated constraints. In each iteration, the algorithm assesses the fitness of the candidate swarm, taking into account both the objective function and constraints. Subsequently, the next generation of candidate swarms is appraised based on the computed fitness values.

By employing the ECO algorithm in these engineering design problems, simultaneous consideration of the objective function and constraints is achieved. This facilitates the search for optimal solutions that satisfy the design requirements and constraints. The effectiveness of the ECO algorithm in handling complex engineering optimization problems is demonstrated through the integration of CHTs. Utilizing the ECO algorithm in engineering design problems enables the simultaneous consideration of both the objective function and constraints, thereby facilitating the pursuit of optimal solutions that meet design specifications and constraints. The effectiveness of the ECO algorithm in addressing intricate engineering optimization challenges is showcased through the incorporation of CHTs.

3.4.1The tension/compression spring design (TSD)

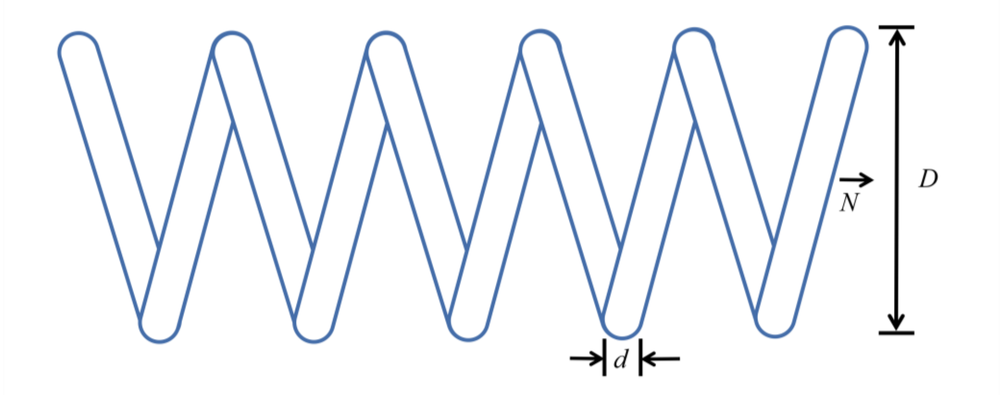


Fig. 11 Schematic illustration of TSD

The primary aim of this challenge is to minimize the weight of the tension/compression spring, emphasizing efficiency in material usage and design optimization [60], as showed in **Fig. 11**. The problem involves three decision variables: wire diameter ($d = x_1$), mean coil diameter ($D = x_2$), and the number of active coils ($N = x_3$). Eq. (14) is formulated to address this optimization challenge, and the outcomes are presented in **Table 15**, alongside the results obtained from other competing algorithms.

Minimize:

$$f(\vec{l}) = (l_3 + 2)l_2 l_1^2 \tag{14}$$

subject to:

$$g_1(\vec{l}) = 1 - \frac{4l_2^2 - l_2 l_3}{717851^4} \leq 0,$$

$$g_2(\vec{l}) = \frac{4l_2^2 - l_1 l_2}{12566(l_3 l_1^3 - l_1^4)} + \frac{1}{5108 l_1^2} \leq 0,$$

$$g_3(\vec{l}) = 1 - \frac{150.45 l_1}{l_2^2 l_3} \leq 0,$$

$$g_4(\vec{l}) = \frac{l_1 + l_2}{1.5} - 1 \leq 0,$$

with bounds:

$$0.05 \leq l_1 \leq 2.00, 0.25 \leq l_2 \leq 1.30, 2.00 \leq l_3 \leq 15.0$$

Table 15 Results of TSD problem

	x1	x2	x3	x4	x5	Best	Worst	Average	STD	Median
ECO	5.19E-02	3.62E-01	1.14E+01	5.09E+02	2.00E+00	1.27E-02	1.32E-02	1.31E-02	3.80E-08	1.32E-02
ALO	5.19E-02	3.62E-01	1.06E+01	4.89E+02	2.00E+00	1.27E-02	1.42E-02	1.32E-02	2.41E-07	1.31E-02
GWO	5.19E-02	3.62E-01	1.07E+01	1.60E+02	2.00E+00	1.27E-02	1.36E-02	1.28E-02	2.74E-08	1.27E-02
WOA	5.27E-02	3.81E-01	9.70E+00	1.00E+03	2.00E+00	1.27E-02	1.78E-02	1.38E-02	1.20E-06	1.36E-02
SSA	6.92E-02	9.41E-01	1.82E+00	8.00E+01	6.00E+01	1.80E-02	3.03E-02	2.28E-02	1.05E-05	2.26E-02
AOA	5.22E-02	3.66E-01	1.09E+01	1.00E+03	2.00E+00	1.30E-02	3.18E-02	1.51E-02	2.99E-05	1.32E-02
HHO	5.27E-02	3.81E-01	9.73E+00	2.33E+02	2.00E+00	1.27E-02	1.67E-02	1.39E-02	9.51E-07	1.36E-02
SCA	5.07E-02	3.32E-01	1.35E+01	1.00E+03	2.00E+00	1.28E-02	1.33E-02	1.32E-02	1.33E-08	1.32E-02
MVO	5.00E-02	3.10E-01	1.50E+01	9.51E+02	2.00E+00	1.32E-02	1.86E-02	1.71E-02	2.91E-06	1.79E-02
ROA	5.37E-02	4.08E-01	8.70E+00	1.00E+03	2.00E+00	1.30E-02	1.63E-02	1.44E-02	8.84E-07	1.43E-02

3.4.2 Gear train design (GTD)

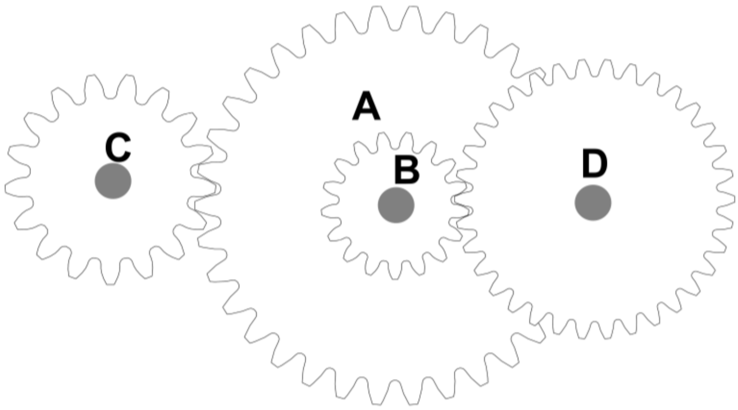


Fig. 12 Schematic illustration of GTD

This subsection verifies the ability of the ECO algorithm to tackle the design problem of gear trains. **Fig. 12** displays a visual illustration of this problem. The goal is to find the number of teeth for each one of the four wheels: A ($= x_1$), B ($= x_2$), C ($= x_3$), and D ($= x_4$) of the gear to minimize the gear ratio. Eq. (15) is formulated to address this optimization challenge, and the outcomes are presented in **Table 16**, alongside the results obtained from other competing algorithms.

Minimize:

$$f(x) = \left(\frac{1}{6.931} - \frac{x_2 x_3}{x_1 x_4}\right)^2 \quad (15)$$

with bounds:

$$12 \le x_1, x_2, x_3, x_4 \le 60.$$

Table 16 Results of GTD problem

	x1	x2	x3	x4	Best	Worst	Average	STD	Median
ECO	4.33E+01	1.64E+01	1.87E+01	4.91E+01	2.70E-12	9.92E-10	5.35E-10	1.73E-19	7.74E-10
ALO	5.65E+01	3.08E+01	1.33E+01	4.94E+01	9.94E-11	2.73E-08	7.76E-09	8.18E-17	4.50E-09
GWO	4.86E+01	1.62E+01	1.90E+01	4.29E+01	2.70E-12	2.36E-09	3.83E-10	3.25E-19	2.31E-11
WOA	4.90E+01	1.94E+01	1.57E+01	4.34E+01	2.70E-12	2.73E-08	2.96E-09	3.25E-17	1.26E-09
SSA	4.91E+01	1.64E+01	1.89E+01	4.31E+01	2.70E-12	1.12E-08	2.12E-09	7.09E-18	9.92E-10
AOA	5.35E+01	2.56E+01	1.47E+01	5.07E+01	2.31E-11	2.73E-08	1.13E-08	1.20E-16	6.35E-09
HHO	4.87E+01	1.93E+01	1.58E+01	4.27E+01	2.70E-12	2.73E-08	2.07E-09	2.38E-17	1.09E-09
SCA	5.33E+01	1.53E+01	2.57E+01	5.12E+01	2.31E-11	1.62E-08	2.49E-09	1.03E-17	1.36E-09
MVO	5.30E+01	1.27E+01	3.02E+01	5.06E+01	2.31E-11	2.36E-09	8.69E-10	5.22E-19	8.89E-10
ROA	3.66E+01	2.33E+01	1.26E+01	5.64E+01	6.60E-10	3.10E-03	1.04E-04	3.11E-07	2.04E-08

3.4.3 The optimal design of an industrial refrigeration system (ODIS)

The mathematical model of this problem is described in [59]. This problem can be formulated as a nonlinear inequality constrained optimization problem, Eq. (16) is formulated. and the outcomes are presented in **Table 17**, alongside the results obtained from other competing algorithms.

Minimize:

$$\begin{aligned} f(\bar{x}) = {} & 63098.88 x_2 x_4 x_{12} + 5441.5 x_2^2 x_{12} + 115055.5 x_2^{1.664} x_6 + 6172.27 x_2^2 x_6 \\ & +63098.88 x_1 x_3 x_{11} + 5441.5 x_1^2 x_{11} + 115055.5 x_1^{1.664} x_5 + 6172.27 x_1^2 x_5 \\ & +140.53 x_1 x_{11} + 281.29 x_3 x_{11} + 70.26 x_1^2 + 281.29 x_1 x_3 + 281.29 x_3^2 \\ & +14437 x_8^{1.8812} x_{12}^{0.3424} x_{10} x_{14}^{-1} x_1^2 x_7 x_9^{-1} + 20470.2 x_7^{2.893} x_{11}^{0.316} x_1^2 \end{aligned} \quad (16)$$

subject to:

$$g_1(\bar{x}) = 1.524 x_7^{-1} \le 0,$$
$$g_2(\bar{x}) = 1.524 x_8^{-1} \le 1,$$
$$g_3(\bar{x}) = 0.07789 x_1 - 2 x_7^{-1} x_9 - 1 \le 0,$$
$$g_4(\bar{x}) = 7.05305 x_9^{-1} x_1^2 x_{10} x_8^{-1} x_2^{-1} x_{14}^{-1} - 1 \le 0,$$
$$g_5(\bar{x}) = 0.0833 x_{13}^{-1} x_{14} - 1 \le 0,$$
$$g_6(\bar{x}) = 47.136 x_2^{0.333} x_{10}^{-1} x_{12} - 1.333 x_8 x_{13}^{2.1195} + 62.08 x_{13}^{2.1195} x_{12}^{-1} x_8^{0.2} x_{10}^{-1} - 1 \le 0,$$
$$g_7(\bar{x}) = 0.04771 x_{10} x_8^{1.8812} x_{12}^{0.3424} - 1 \le 0,$$

$$g_8(\bar{x}) = 0.0488 x_9 x_7^{1.893} x_{11}^{0.316} - 1 \leq 0,$$
$$g_9(\bar{x}) = 0.0099 x_1 x_3^{-1} - 1 \leq 0,$$
$$g_{10}(\bar{x}) = 0.0193 x_2 x_4^{-1} - 1 \leq 0,$$
$$g_{11}(\bar{x}) = 0.0298 x_1 x_5^{-1} - 1 \leq 0,$$
$$g_{12}(\bar{x}) = 0.056 x_2 x_6^{-1} - 1 \leq 0,$$
$$g_{13}(\bar{x}) = 2 x_9^{-1} - 1 \leq 0,$$
$$g_{14}(\bar{x}) = 2 x_{10}^{-1} - 1 \leq 0,$$
$$g_{15}(\bar{x}) = x_{12} x_{11}^{-1} - 1 \leq 0$$

with bounds:

$$0.001 \leq x_i \leq 5, i = 1, \ldots, 14 \; i = 1, \ldots, 14.$$

Table 17 Results of ODIS problem

	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	Best	Worst	Average	STD	Median
ECO	1.00E-03	1.00E-03	1.00E-03	1.00E-03	1.00E-03	1.00E-03	1.55E+00	1.52E+00	4.97E+00	4.97E+00	2.86E-01	1.16E-01	3.74E-02	2.79E-01	2.85E-01	2.12E+01	5.73E+00	3.75E+01	2.91E+00
ALO	1.00E-03	1.00E-03	1.00E-03	8.07E-01	1.00E-03	1.00E-03	1.62E+00	3.58E+00	4.05E+00	2.09E+00	1.00E-03	1.00E-03	6.60E-03	7.92E-02	2.21E-01	8.78E+20	1.18E+20	3.67E+40	9.64E+19
GWO	1.00E-03	1.01E-03	1.07E-03	2.87E-03	1.60E-03	2.82E-03	1.53E+00	1.53E+00	4.99E+00	2.00E+00	2.86E-03	2.75E-03	1.17E-02	1.39E-01	3.76E-02	9.87E+19	1.27E+19	1.04E+39	5.58E-02
WOA	1.00E-03	1.00E-03	1.00E-03	1.46E+00	1.10E-03	2.20E-03	1.52E+00	1.53E+00	5.00E+00	2.22E+00	1.57E-03	1.37E-03	2.02E-03	1.57E-02	2.85E-01	3.57E+21	6.32E+20	1.41E+42	4.91E+06
SSA	1.00E-03	1.00E-03	1.00E-03	1.01E+00	9.75E-01	2.21E-01	2.62E+00	4.02E+00	4.64E+00	2.05E+00	9.72E-02	5.30E-03	1.47E-02	1.77E-01	2.17E+00	1.01E+20	3.35E+18	3.26E+38	2.69E+01
AOA	1.00E-03	1.00E-03	1.00E-03	2.70E+00	1.80E+00	4.64E-01	1.79E+00	2.73E+00	5.00E+00	5.00E+00	3.37E-03	1.00E-03	5.16E-03	1.32E-02	4.07E+00	2.40E+20	7.06E+19	4.19E+39	9.31E+19
HHO	1.00E-03	1.00E-03	1.00E-03	1.98E+00	1.00E-03	1.00E-03	1.98E+00	1.53E+00	2.01E+00	2.00E+00	1.00E-03	1.00E-03	6.35E-03	7.28E-02	2.27E-01	7.48E+20	4.40E+19	1.89E+40	1.86E+03
SCA	1.01E-03	1.00E-03	1.63E-03	5.21E-03	1.47E-03	1.25E-03	1.99E+00	1.55E+00	5.00E+00	2.48E+00	9.78E-03	3.19E-03	1.27E-02	1.02E-01	9.29E-02	3.49E-01	2.01E-01	4.21E-03	1.90E-01
MVO	1.00E-03	1.00E-03	1.00E-03	1.00E-03	1.00E-03	1.00E-03	2.08E+00	2.14E+00	2.53E+00	4.55E+00	2.14E-01	2.62E-02	4.80E-02	5.64E-01	3.30E-01	9.38E+19	2.50E+19	1.72E+39	5.46E+00
ROA	5.63E-02	3.09E-02	1.61E-01	4.95E+00	2.24E+00	1.32E-01	8.13E-01	2.68E+00	4.95E+00	4.95E+00	2.77E+00	4.95E+00	4.95E+00	4.95E+00	3.96E+20	6.39E+21	3.88E+21	1.60E+42	4.34E+21

3.4.4 Multiple disk clutch brake design (MDCBD)

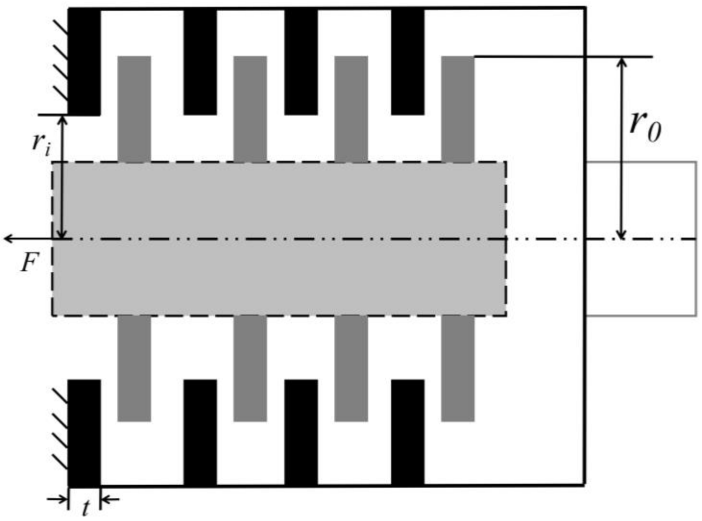


Fig. 13 Schematic illustration of MDCBD

The multiple disk clutch brake design problem aims to minimize the mass of the multiple disk clutch brake, as defined by Eq. (13). This problem is characterized by nine nonlinear constraints and involves five discrete design variables, namely the inner radius (x_1), outer radius (x_2), disk thickness (x_3), actuator force (x_4), and number of frictional surfaces (x_5). The schematic representation of this problem is depicted in **Fig. 13**. The results of optimization algorithms applied to this problem are tabulated in **Table 18**, revealing the superior performance of the ECO algorithm in comparison to other algorithms under consideration.

Minimize:

$$f(\bar{x}) = \pi(x_2^2 - x_1^2)x_3(x_5 + 1)\rho \tag{17}$$

subject to:

$$g_1(\bar{x}) = -p_{max} + p_{rz} \le 0,$$

$$g_2(\bar{x}) = p_{rz}V_{sr} - V_{sr,max}p_{max} \le 0,$$

$$g_3(\bar{x}) = \Delta R + x_1 - x_2 \le 0,$$

$$g_4(\bar{x}) = -L_{max} + (x_5 + 1)(x_3 + \delta) \le 0,$$

$$g_5(\bar{x}) = sM_s - M_{\hbar} \le 0,$$

$$g_6(\bar{x}) = T \ge 0,$$

$$g_7(\bar{x}) = -V_{sr,max} + V_{sr} \le 0,$$

$$g_7(\bar{x}) = T - T_{max} \le 0,$$

where,

$$M_{\hbar} = \frac{2}{3}\mu x_4 x_5 \frac{x_2^3 - x_1^3}{x_2^2 - x_1^2} N.mm,$$

$$\omega = \frac{\pi n}{30} rad/s,$$

$$A = \pi(x_2^2 - x_1^2)mm^2,$$

$$p_{rz} = \frac{x_4}{A} N/mm^2,$$

$$V_{sr} = \frac{\pi R_{sr} n}{30} mm/s,$$

$$R_{sr} = \frac{2}{3}\frac{x_2^3 - x_1^3}{x_2^2 x_1^2} mm,$$

$$T = \frac{I_z \omega}{M_h + M_f},$$

$$\Delta R = 20mm, L_{max} = 30mm, L_{max} = 30mm, \mu=0.6,$$

$$V_{sr,max} = 10\, m/s, \delta = 0.5mm, s=1.5,$$

$$T_{max} = 15s, n = 250rpm, I_z = 55Kg \cdot m^2,$$

$$M_s = 40Nm, M_f = 3Nm, \text{ and } p_{max} = 1$$

with bounds:

$$60 \le x_1 \le 80, 90 \le x_2 \le 110, 1 \le x_3 \le 3$$

$$0 \le x_4 \le 1000, 2 \le x_5 \le 9.$$

Table 18 Results of MDCBD problem

	x1	x2	x3	x4	x5	Best	Worst	Average	STD	Median
ECO	7.00E+01	9.00E+01	1.00E+00	2.72E+02	2.00E+00	2.35E-01	2.35E-01	2.35E-01	3.30E-14	2.35E-01
ALO	7.00E+01	9.00E+01	1.00E+00	4.89E+02	2.00E+00	2.35E-01	2.35E-01	2.35E-01	8.66E-17	2.35E-01
GWO	7.00E+01	9.00E+01	1.00E+00	1.60E+02	2.00E+00	2.35E-01	2.36E-01	2.35E-01	5.00E-09	2.35E-01
WOA	7.00E+01	9.00E+01	1.00E+00	1.00E+03	2.00E+00	2.35E-01	2.35E-01	2.35E-01	1.95E-14	2.35E-01
SSA	6.00E+01	8.00E+01	6.00E+01	8.00E+01	6.00E+01	1.34E+27	1.34E+27	1.34E+27	0.00E+00	1.34E+27
AOA	7.00E+01	9.00E+01	1.00E+00	1.00E+03	2.00E+00	2.35E-01	2.66E-01	2.40E-01	6.78E-05	2.36E-01
HHO	7.00E+01	9.00E+01	1.00E+00	2.33E+02	2.00E+00	2.35E-01	2.35E-01	2.35E-01	2.59E-17	2.35E-01
SCA	7.00E+01	9.00E+01	1.00E+00	1.00E+03	2.00E+00	2.35E-01	2.40E-01	2.37E-01	1.48E-06	2.37E-01
MVO	7.00E+01	9.00E+01	1.00E+00	9.51E+02	2.00E+00	2.35E-01	2.53E-01	2.36E-01	1.04E-05	2.35E-01
ROA	7.00E+01	9.00E+01	1.00E+00	1.00E+03	2.00E+00	2.35E-01	3.31E-01	3.04E-01	9.43E-04	3.15E-01

3.4.5 Speed reducer design (SRD)

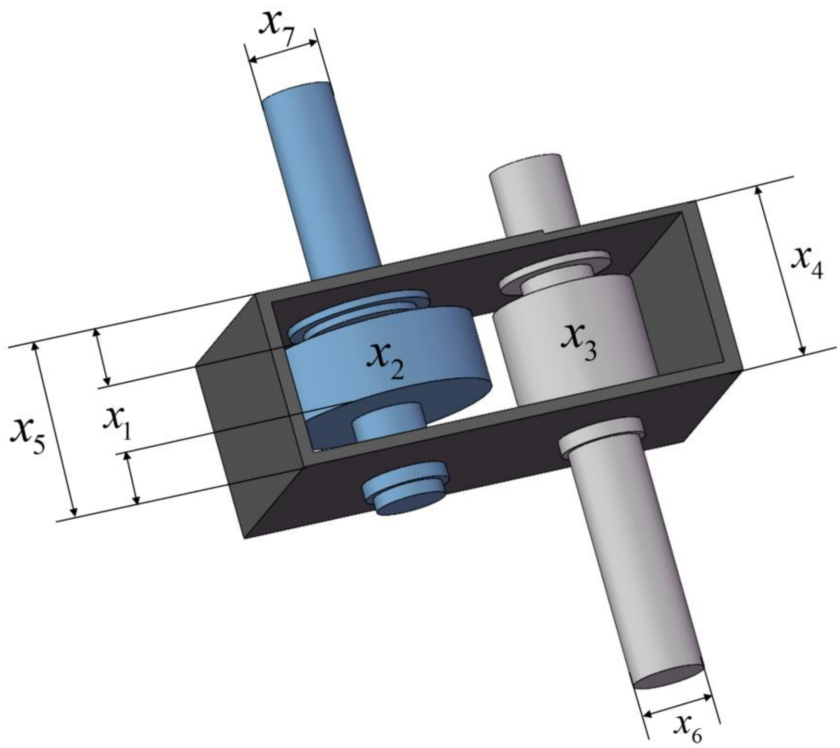


Fig. 14 Schematic illustration of SRD

The primary aim of the SRD problem, classified as a discrete challenge, is to identify the optimal weight for the speed reducer while adhering to four essential design constraints. These constraints encompass the bending stress of the gear teeth, covering stress, transverse deflections of the shafts, and stresses within the shafts, all depicted in **Fig. 14**. Consequently, the problem involves managing one discrete variable and six continuous variables. Specifically, x_1 signifies the face width, x_2 represents the module of teeth, and x_3 denotes a discrete design variable pertaining to the arrangement of teeth in the pinion. Correspondingly, x_4 signifies the length of the first shaft between bearings, while x_5 pertains to the length of the second shaft between bearings. The sixth and seventh design variables (x_6 and x_7) correspond to the diameters of the first and second shaft, respectively. The mathematical formulation of this task is as follows. **Table 19** reports the results obtained by optimization algorithms for this problem, in which ECO algorithms outperform other comparative algorithms.

Minimize:

$$f(x) = 0.7854x_2^2x_1(14.9334x_3 - 43.0934 + 3.3333x_3^2) + 0.7854(x_5x_7^2 + x_4x_6^2) - 1.508x_1(x_7^2 + x_6^2) + 7.477(x_7^2 + x_6^2) \quad (18)$$

subject to:

$$g_1(x) = -x_1x_2^2x_3 + 27 \leq 0,$$
$$g_2(x) = -x_1x_2^2x_3 + 397.5 \leq 0,$$
$$g_3(x) = -x_2x_6^4x_3x_4^{-3} + 1.93 \leq 0,$$
$$g_4(x) = -x_2x_7^4x_3x_5^{-3} + 1.93 \leq 0,$$

$$g_5(x) = 10x_6^{-3}\sqrt{16.91 \times 10^6 + (745x_4x_2^{-1}x_3^{-1})^2} - 1100 \leq 0,$$

$$g_6(x) = 10x_7^{-3}\sqrt{157.5 \times 10^6 + (745x_5x_2^{-1}x_3^{-1})^2} - 850 \leq 0,$$

$$g_7(x) = x_2x_3 - 40 \leq 0,$$
$$g_8(x) = -x_1x_2^{-1} + 5 \leq 0,$$
$$g_9(x) = x_1x_2^{-1} - 12 \leq 0,$$
$$g_{10}(x) = 1.5x_6 - x_4 + 1.9 \leq 0,$$
$$g_{11}(x) = 1.1x_7 - x_5 + 1.9 \leq 0,$$

where,

$$2.6 \leq x_1 \leq 3.6, 0.7 \leq x_2 \leq 0.8, 17 \leq x_3 \leq 28,$$
$$x_4 \leq 8.3, 7.3 \leq x_5, 2.9 \leq x_6 \leq 3.9, 5 \leq x_7 \leq 5.5,$$

Table 19 Results of SRD problem

	x1	x2	x3	x4	x5	x6	x7	Best	Worst	Average	STD	Median
ECO	3.50E+00	7.00E-01	1.70E+01	7.33E+00	7.72E+00	3.35E+00	5.29E+00	2.99E+03	3.04E+03	3.01E+03	1.43E+02	3.01E+03
ALO	3.50E+00	7.00E-01	1.70E+01	7.40E+00	7.72E+00	3.35E+00	5.29E+00	3.00E+03	3.02E+03	3.00E+03	1.76E+01	3.01E+03
GWO	3.50E+00	7.00E-01	1.70E+01	7.37E+00	7.84E+00	3.35E+00	5.29E+00	3.00E+03	3.03E+03	3.01E+03	2.80E+01	3.01E+03
WOA	3.50E+00	7.00E-01	1.70E+01	7.98E+00	8.03E+00	3.38E+00	5.29E+00	3.01E+03	5.32E+03	3.26E+03	1.97E+05	3.13E+03
SSA	3.60E+00	3.60E+00	3.60E+00	3.60E+00	2.60E+00	3.35E+00	3.60E+00	3.39E+26	3.39E+26	3.39E+26	3.43E+39	3.39E+26
AOA	3.60E+00	7.00E-01	1.70E+01	8.30E+00	8.30E+00	3.36E+00	5.30E+00	3.07E+03	3.23E+03	3.16E+03	1.96E+03	3.17E+03
HHO	3.53E+00	7.00E-01	1.70E+01	8.04E+00	8.07E+00	3.35E+00	5.30E+00	3.03E+03	5.37E+03	3.66E+03	4.08E+05	3.49E+03
SCA	3.57E+00	7.00E-01	1.70E+01	7.60E+00	8.23E+00	3.39E+00	5.32E+00	3.07E+03	3.20E+03	3.14E+03	1.73E+03	3.14E+03
MVO	3.50E+00	7.00E-01	1.70E+01	7.90E+00	8.27E+00	3.36E+00	5.29E+00	3.02E+03	3.08E+03	3.05E+03	1.84E+02	3.05E+03
ROA	3.52E+00	7.00E-01	1.70E+01	8.12E+00	8.12E+00	3.49E+00	5.29E+00	3.06E+03	2.50E+19	2.17E+18	3.62E+37	3.26E+03

3.4.6 Rolling element bearing design (REBD)

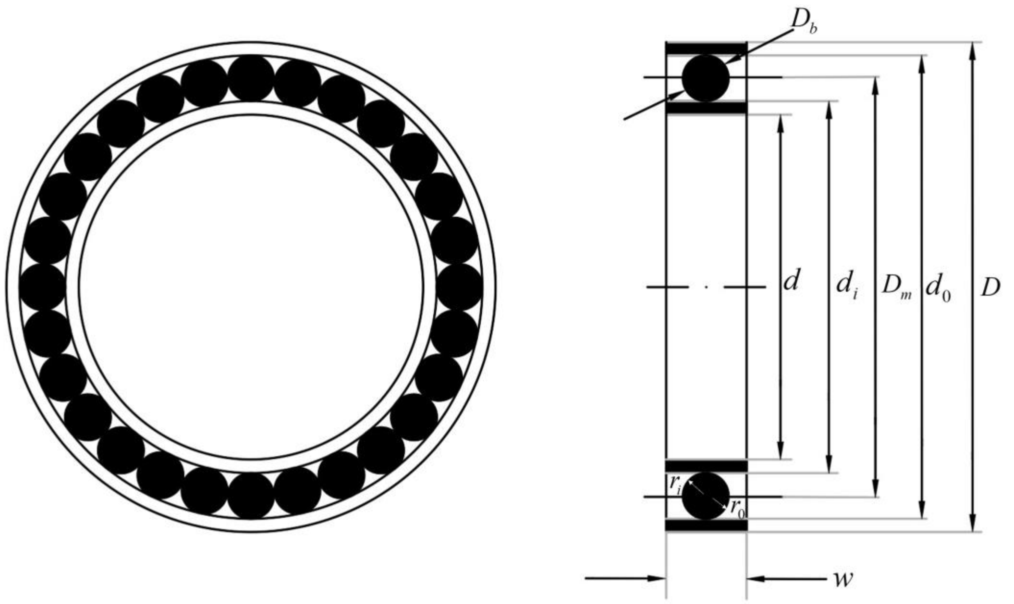


Fig. 15 Schematic illustration of REBD

This engineering problem involves 10 geometric variables and considers nine assembly constraints along with geometric-based limitations. Our objective in addressing this scenario is to optimize (maximize) the dynamic load-carrying capacity. The formulation of this test case is outlined below. The schematic of this problem is shown in **Fig. 15**. **Table 20** reports the obtained results of optimization algorithms for this problem in which ECO algorithms outperform other comparative algorithms.

Maximize:

$$f(\bar{x}) = \begin{cases} f_c Z^{\frac{2}{3}} D_b^{1.8} \ \ if \ \ D_b \leq 25.4\text{mm} \\ 3.647 f_c Z^{\frac{2}{3}} D_b^{1.4} \text{ ,otherwise} \end{cases} \tag{19}$$

where:

$$g_1(\bar{x}) = Z - \frac{\phi_0}{2\, sin^{-1}\left(\frac{D_b}{D_m}\right)} - 1 \leq 0,$$
$$g_2(\bar{x}) = K_{Dmin}(D - d) - 2D_b \leq 0,$$
$$g_3(\bar{x}) = 2D_b - K_{Dmax}(D - d) \leq 0,$$
$$g_4(\bar{x}) = D_b - w \leq 0,$$
$$g_5(\bar{x}) = 0.5(D + d) - D_m \leq 0,$$
$$g_6(\bar{x}) = D_m - (0.5 + e)(D + d) \leq 0,$$
$$g_7(\bar{x}) = \xi D_b - 0.5(D - D_m - D_b) \leq 0,$$
$$g_8(\bar{x}) = 0.515 - f_i \leq 0,$$
$$g_9(\bar{x}) = 0.515 - f_0 \leq 0,$$

where,

$$f_c = 37.91\left\{1+\left\{1.04\left(\frac{1-\gamma}{1+\gamma}\right)^{1.72}\left(\frac{f_i(2f_0-1)}{f_0(2f_i-1)}\right)^{0.41}\right\}^{\frac{10}{3}}\right\}^{-0.3}, \gamma = \frac{D_b\,cos(\alpha)}{D_m}, f_i = \frac{r_i}{D_b}, f_0 = \frac{r_0}{D_b}$$

$$\phi_0 = 2\pi - 2\times cos^{-1}\left(\frac{\left\{\frac{(D-d)}{2-3\left(\frac{T}{4}\right)}\right\}^2 + \left\{\frac{D}{2}-\left(\frac{T}{4}\right)-D_b\right\}^2 - \left\{\frac{d}{2+\left(\frac{T}{4}\right)}\right\}^2}{2\left\{\frac{(D-d)}{2}-3\left(\frac{T}{4}\right)\right\}\left\{\frac{D}{2}-\left(\frac{T}{4}\right)-D_b\right\}}\right)$$

$$T = D = d - 2D_b, D = 160, d = 90, B_w = 30.$$

with bounds:

$$0.5(D+d) \le D_m \le 0.6(D+d),$$
$$0.15(D-d) \le D_b \le 0.45(D-d),$$
$$4 \le Z \le 50,$$
$$0.515 \le f_i \le 0.6,$$
$$0.515 \le f_0 \le 0.6,$$
$$0.4 \le K_{Dmin} \le 0.5,$$
$$0.6 \le K_{Dmax} \le 0.7,$$
$$0.3 \le \xi \le 0.4,$$
$$0.02 \le e \le 0.1,$$
$$0.6 \le \zeta \le 0.85,$$

Table 20 Results of REBD problem

	x2	x3	x4	x5	x6	x7	x8	x9	x10	Best	Worst	Average	STD	Median
+0	2.14E+01	1.06E+01	5.15E-01	5.89E-01	4.98E-01	6.98E-01	3.02E-01	9.03E-02	6.35E-01	-8.52E+04	-8.35E+04	-8.45E+04	5.72E+04	-8.45E+04
+0	2.14E+01	1.14E+01	5.15E-01	5.15E-01	4.00E-01	6.19E-01	3.00E-01	9.96E-02	6.00E-01	-8.55E+04	-8.46E+04	-8.54E+04	5.54E+04	-8.55E+04
+0	2.14E+01	1.10E+01	5.15E-01	5.64E-01	4.86E-01	6.95E-01	3.00E-01	2.26E-02	6.18E-01	-8.55E+04	-8.47E+04	-8.53E+04	2.84E+04	-8.54E+04
+0	2.14E+01	1.09E+01	5.15E-01	5.25E-01	4.10E-01	6.11E-01	3.08E-01	3.15E-02	6.44E-01	-8.51E+04	-4.52E+04	-7.67E+04	1.05E+08	-8.17E+04
+0	1.25E+02	1.50E+02	1.50E+02	1.25E+02	1.50E+02	1.25E+02	1.50E+02	1.25E+02	1.25E+02	6.55E+04	6.55E+04	6.55E+04	0.00E+00	6.55E+04
+0	2.19E+01	9.61E+00	5.15E-01	5.61E-01	4.00E-01	6.45E-01	3.00E-01	2.42E-02	6.00E-01	-8.32E+04	-6.87E+04	-7.59E+04	1.66E+07	-7.68E+04
+0	2.14E+01	1.13E+01	5.15E-01	5.15E-01	4.38E-01	6.25E-01	3.00E-01	5.63E-02	6.66E-01	-8.55E+04	-4.23E+04	-6.80E+04	2.25E+08	-6.98E+04
+0	2.12E+01	1.12E+01	5.15E-01	5.15E-01	4.00E-01	7.00E-01	3.00E-01	2.00E-02	6.00E-01	-8.36E+04	-7.12E+04	-7.90E+04	7.82E+06	-7.91E+04
+0	2.14E+01	1.08E+01	5.15E-01	5.20E-01	4.46E-01	6.75E-01	3.00E-01	9.29E-02	6.78E-01	-8.55E+04	-7.85E+04	-8.48E+04	2.82E+06	-8.54E+04
+0	2.10E+01	1.12E+01	5.15E-01	5.15E-01	4.00E-01	6.00E-01	3.00E-01	5.42E-02	6.00E-01	-8.25E+04	-7.08E+04	-8.07E+04	9.96E+06	-8.21E+04

3.4.7 The ECO algorithm is good at solving real-world problems

Table 21 Rank of real-world problem

	ECO	ALO	GWO	WOA	SSA	AOA	HHO	SCA	MVO	ROA
TSD	2	4	1	5	10	8	6	3	9	7
GTD	2	8	1	7	5	9	4	6	3	10
ODIS	2	8	4	9	3	7	6	1	5	10
MDCBD	1	3	5	4	10	8	2	7	6	9
SRD	2	1	3	7	10	6	8	5	4	9
REBD	4	1	2	7	10	8	9	6	3	5

Average Rank	2.17	4.17	2.67	6.50	8.00	7.67	5.83	4.67	5.00	8.33
Final Ranking	1	3	2	7	9	8	6	4	5	10

Table 21 presents the performance evaluation of the ECO algorithm and nine existing frontier algorithms on six real-world engineering problems. The outcomes unequivocally indicate that the ECO algorithm outperforms the other nine optimization algorithms and exhibits superior stability. These findings demonstrate the compelling competitiveness of the ECO algorithm in regard to solving optimization problems with realistic constraints. The algorithm's consistently delivering excellent results further underscores its potential as an effective and robust optimization tool for real-world applications.

3.5 Strengths and limitations of the ECO

As highlighted in the paper, the proposed ECO algorithm demonstrates significant theoretical potential in solving a wide range of optimization problems and surpasses nine state-of-the-art algorithms. ECO adopts a simulation of intense competition in education by categorizing populations into schools and students. It performs exceptionally well in handling single-peak, multipeak, and hybrid functions and in solving practical real-world problems.

It's essential to acknowledge that, like many other algorithms, ECO does not guarantee the computation of an optimal solution. Particularly, when the initial solution is already close to the optimal solution (e.g., cases F21-F23), the ECO algorithm may exhibit a lower convergence rate, necessitating more iterations to obtain a better solution. Future research should prioritize further evaluating the performance of ECO on real-world problems, which will offer valuable insights and contribute to its continuous enhancement.

Moreover, the experimental findings suggest that ECO may occasionally converge to local optimal solutions within a limited set of functions, as exemplified by F13 among the 23 classical test functions. Addressing this issue warrants further investigation in our upcoming research endeavors.

4 Conclusions and future works

In conclusion, the Educational Competition Optimizer (ECO) presented in this paper has demonstrated its effectiveness as a meta-heuristic algorithm inspired by educational competition. ECO models the progressive competitiveness of students through three stages: elementary, middle, and high school, effectively mirroring the process of acquiring a better education. The algorithm classifies students into two categories based on learning patience and employs an alternating stage strategy to achieve rapid convergence, demonstrating its potential for solving optimization problems, particularly in constrained engineering design scenarios.

In the experiments we conducted, we initially performed a comprehensive parameter sensitivity analysis to optimize ECO's performance, revealing that a population size of 40 offers the highest performance. Interestingly, the population size does not correlate linearly with convergence ability. Subsequently, ECO was rigorously compared with nine state-of-the-art algorithms across a diverse set of functions encompassing single-peak, multimodal, hybrid, and combined functions. The results showcased ECO's superior ability to balance exploration and exploitation, positioning it as a robust contender among its peers. Moreover, ECO exhibited its prowess by successfully addressing five engineering optimization problems. In comparative evaluations against nine renowned algorithms, ECO consistently outperformed or held its ground in five selected performance metrics. This robust and efficient performance can be theoretically attributed to three key factors: 1) A constant alternation of exploration and exploitation through a strategy involving three educational competition phases for efficient convergence. 2) A multi-strategy search methodology ensuring algorithmic stochasticity and population diversity. 3) Drawing inspiration from merit-based educational competition to eliminate low-quality solutions from the search population, effectively reducing performance degradation.

While ECO has demonstrated its potential, areas warranting further investigation in future research remain. Striking the optimal balance between the three phases of ECO presents a significant challenge but can potentially enhance algorithm efficiency and performance. Besides, the application of ECO can also be extended to other domains such as energy [61], image segmentation [62], the Internet of Things [63], or scheduling problems [64], thereby enhancing its universality. Moreover, delving into hybrid methodologies that fuse ECO with other well-established metaheuristics shows great potential. These efforts could yield more robust and adaptable optimization techniques, unlocking novel avenues and potentially propelling the field of optimization forward significantly.

·CRediT authorship contribution statement

Junbo Lian: Conceptualization, Investigation, Methodology, Resources, Software, Formal analysis, Data curation, Visualization, Writing - Original Draft, Writing - Review & Editing.

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Ling Ma: Data curation, Validation.
Xincan Wu: Visualization.
Ali Asghar Heidari: Formal analysis, Resources, Investigation, Writing - Review & Editing.
Yi Chen: Investigation, Writing - Review & Editing.
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Guohua Hui: Supervision, Funding acquisition, Project administration, Writing - Review & Editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data Availability Statement

Data will be made available on request.

References

[1] X. Li and Y. Sun, "Application of RBF neural network optimal segmentation algorithm in credit rating," *Neural Computing and Applications,* vol. 33, pp. 8227-8235, 2021.
[2] H. Faris, I. Aljarah, M. A. Al-Betar, S. J. N. c. Mirjalili, and applications, "Grey wolf optimizer: a review of recent variants and applications," vol. 30, pp. 413-435, 2018.
[3] X.-S. J. M. i. w. Yang, geotechnical and t. engineering, "Optimization and metaheuristic algorithms in engineering," vol. 1, p. 23, 2013.
[4] V. Kumar, J. K. Chhabra, and D. J. J. o. C. S. Kumar, "Parameter adaptive harmony search algorithm for unimodal and multimodal optimization problems," vol. 5, no. 2, pp. 144-155, 2014.
[5] C. Wang, J. M. Koh, T. Yu, N. G. Xie, K. H. J. C. M. i. A. M. Cheong, and Engineering, "Material and shape optimization of bi-directional functionally graded plates by GIGA and an improved multi-objective particle swarm optimization algorithm," vol. 366, p. 113017, 2020.
[6] W. Zhao, C. Du, S. J. C. M. i. A. M. Jiang, and Engineering, "An adaptive multiscale approach for identifying multiple flaws based on XFEM and a discrete artificial fish swarm algorithm," vol. 339, pp. 341-357, 2018.
[7] J. Zhang, M. Xiao, L. Gao, and Q. J. A. M. M. Pan, "Queuing search algorithm: A novel metaheuristic algorithm for solving engineering optimization problems," vol. 63, pp. 464-490, 2018.
[8] C. Zhang, L. Zhou, and Y. Li, "Pareto optimal reconfiguration planning and distributed parallel motion control of mobile modular robots," *IEEE Transactions on Industrial Electronics,* 2023.
[9] B. Cao, J. Zhao, Y. Gu, Y. Ling, and X. Ma, "Applying graph-based differential grouping for multiobjective large-scale optimization," *Swarm and Evolutionary Computation,* vol. 53, p. 100626, 2020.
[10] B. Cao *et al.*, "Multiobjective 3-D Topology Optimization of Next-Generation Wireless Data Center Network," *IEEE Transactions on Industrial Informatics,* vol. 16, no. 5, pp. 3597-3605, 2019.
[11] Y. Duan, Y. Zhao, and J. Hu, "An initialization-free distributed algorithm for dynamic economic dispatch problems in microgrid: Modeling, optimization and analysis," *Sustainable Energy, Grids and Networks,* p. 101004, 2023.
[12] B. Cao, J. Zhao, Y. Gu, S. Fan, and P. Yang, "Security-aware industrial wireless sensor network deployment optimization," *IEEE Transactions on Industrial Informatics,* vol. 16, no. 8, pp. 5309-5316, 2019.
[13] Y. Xie, X. Y. Wang, Z. J. Shen, Y. H. Sheng, and G. X. Wu, "A Two-Stage Estimation of Distribution Algorithm With Heuristics for Energy-Aware Cloud Workflow Scheduling," *IEEE Transactions on Services Computing,* vol. 16, no. 6, pp. 4183-4197, 2023, doi: 10.1109/TSC.2023.3311785.
[14] J. Mou, K. Gao, P. Duan, J. Li, A. Garg, and R. Sharma, "A Machine Learning Approach for Energy-Efficient Intelligent Transportation Scheduling Problem in a Real-World Dynamic Circumstances," *IEEE Transactions on Intelligent Transportation Systems,* vol. 24, no. 12, pp. 15527-15539, 2023, doi: 10.1109/TITS.2022.3183215.

[15] S. Gholizadeh, M. Danesh, C. J. C. Gheyratmand, and Structures, "A new Newton metaheuristic algorithm for discrete performance-based design optimization of steel moment frames," vol. 234, p. 106250, 2020.
[16] D. Tang, S. Dong, Y. Jiang, H. Li, and Y. J. A. S. C. Huang, "ITGO: Invasive tumor growth optimization algorithm," vol. 36, pp. 670-698, 2015.
[17] T. Lyu, H. Xu, L. Zhang, and Z. Han, "Source Selection and Resource Allocation in Wireless-Powered Relay Networks: An Adaptive Dynamic Programming-Based Approach," *IEEE Internet of Things Journal,* vol. 11, no. 5, pp. 8973-8988, 2024, doi: 10.1109/JIOT.2023.3321673.
[18] G. Sun, Y. Zhang, H. Yu, X. Du, and M. Guizani, "Intersection fog-based distributed routing for V2V communication in urban vehicular ad hoc networks," *IEEE Transactions on Intelligent Transportation Systems,* vol. 21, no. 6, pp. 2409-2426, 2019.
[19] R. Moghdani and K. J. A. S. C. Salimifard, "Volleyball premier league algorithm," vol. 64, pp. 161-185, 2018.
[20] D. B. Fogel, *Artificial intelligence through simulated evolution*. Wiley-IEEE Press, 1998.
[21] G. Sun, Y. Zhang, D. Liao, H. Yu, X. Du, and M. Guizani, "Bus-trajectory-based street-centric routing for message delivery in urban vehicular ad hoc networks," *IEEE Transactions on Vehicular Technology,* vol. 67, no. 8, pp. 7550-7563, 2018.
[22] E. H. Houssein, D. Oliva, N. A. Samee, N. F. Mahmoud, and M. M. Emam, "Liver Cancer Algorithm: A novel bio-inspired optimizer," *Computers in Biology and Medicine,* vol. 165, p. 107389, 2023/10/01/ 2023, doi: https://doi.org/10.1016/j.compbiomed.2023.107389.
[23] R. Storn and K. J. J. o. g. o. Price, "Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces," vol. 11, pp. 341-359, 1997.
[24] N. Hansen, S. D. Müller, and P. J. E. c. Koumoutsakos, "Reducing the time complexity of the derandomized evolution strategy with covariance matrix adaptation (CMA-ES)," vol. 11, no. 1, pp. 1-18, 2003.
[25] S. Shadravan, H. R. Naji, and V. K. J. E. A. o. A. I. Bardsiri, "The Sailfish Optimizer: A novel nature-inspired metaheuristic algorithm for solving constrained engineering optimization problems," vol. 80, pp. 20-34, 2019.
[26] S. Li, H. Chen, Y. Chen, Y. Xiong, and Z. Song, "Hybrid Method with Parallel-Factor Theory, a Support Vector Machine, and Particle Filter Optimization for Intelligent Machinery Failure Identification," *Machines,* vol. 11, no. 8, p. 837, 2023.
[27] H. Su *et al.*, "RIME: A physics-based optimization," vol. 532, pp. 183-214, 2023.
[28] J. Tu, H. Chen, M. Wang, and A. H. Gandomi, "The Colony Predation Algorithm," *Journal of Bionic Engineering,* vol. 18, no. 3, pp. 674-710, 2021/05/01 2021, doi: 10.1007/s42235-021-0050-y.
[29] A. A. Heidari, S. Mirjalili, H. Faris, I. Aljarah, M. Mafarja, and H. J. F. g. c. s. Chen, "Harris hawks optimization: Algorithm and applications," vol. 97, pp. 849-872, 2019.
[30] H. Chen, C. Li, M. Mafarja, A. A. Heidari, Y. Chen, and Z. Cai, "Slime mould algorithm: a comprehensive review of recent variants and applications," *International Journal of Systems Science,* pp. 1-32, 2022.
[31] S. Li, H. Chen, M. Wang, A. A. Heidari, and S. Mirjalili, "Slime mould algorithm: A new method for stochastic optimization," *Future Generation Computer Systems,* vol. 111, pp. 300-323, 2020/04/03/ 2020, doi: https://doi.org/10.1016/j.future.2020.03.055.
[32] S. Mirjalili and A. J. A. i. e. s. Lewis, "The whale optimization algorithm," vol. 95, pp. 51-67, 2016.
[33] I. Ahmadianfar, A. Asghar Heidari, S. Noshadian, H. Chen, and A. H. Gandomi, "INFO: An Efficient Optimization Algorithm based on Weighted Mean of Vectors," *Expert Systems with Applications,* p. 116516, 2022/01/15/ 2022, doi: https://doi.org/10.1016/j.eswa.2022.116516.
[34] J. Lian *et al.*, "Parrot optimizer: Algorithm and applications to medical problems," *Computers in Biology and Medicine,* p. 108064, 2024/02/24/ 2024, doi: https://doi.org/10.1016/j.compbiomed.2024.108064.
[35] Y. Yang, H. Chen, A. A. Heidari, and A. H. Gandomi, "Hunger games search: Visions, conception, implementation, deep analysis, perspectives, and towards performance shifts," *Expert Systems with Applications,* vol. 177, p. 114864, 2021/09/01/ 2021, doi: https://doi.org/10.1016/j.eswa.2021.114864.
[36] E. Rashedi, H. Nezamabadi-Pour, and S. J. I. s. Saryazdi, "GSA: a gravitational search algorithm," vol. 179, no. 13, pp. 2232-2248, 2009.
[37] S. Mirjalili, S. M. Mirjalili, A. J. N. C. Hatamlou, and Applications, "Multi-verse optimizer: a nature-inspired

algorithm for global optimization," vol. 27, pp. 495-513, 2016.
[38] A. Kaveh and S. J. A. m. Talatahari, "A novel heuristic optimization method: charged system search," vol. 213, no. 3-4, pp. 267-289, 2010.
[39] B. Zhu, Y. Sun, J. Zhao, J. Han, P. Zhang, and T. Fan, "A Critical Scenario Search Method for Intelligent Vehicle Testing Based on the Social Cognitive Optimization Algorithm," *IEEE Transactions on Intelligent Transportation Systems,* 2023.
[40] E. Atashpaz-Gargari and C. Lucas, "Imperialist competitive algorithm: an algorithm for optimization inspired by imperialistic competition," in *2007 IEEE congress on evolutionary computation*, 2007: Ieee, pp. 4661-4667.
[41] J. Lian and G. Hui, "Human evolutionary optimization algorithm," *Expert Systems with Applications,* vol. 241, p. 122638, 2024.
[42] Z. Xiao *et al.*, "Multi-objective parallel task offloading and content caching in D2D-aided MEC networks," *IEEE Transactions on Mobile Computing,* 2022.
[43] D. H. Wolpert and W. G. J. I. t. o. e. c. Macready, "No free lunch theorems for optimization," vol. 1, no. 1, pp. 67-82, 1997.
[44] X. Yao, Y. Liu, and G. J. I. T. o. E. c. Lin, "Evolutionary programming made faster," vol. 3, no. 2, pp. 82-102, 1999.
[45] A. W. Mohamed, A. A. Hadi, P. Agrawal, K. M. Sallam, and A. K. Mohamed, "Gaining-sharing knowledge based algorithm with adaptive parameters hybrid with IMODE algorithm for solving CEC 2021 benchmark problems," in *2021 IEEE Congress on Evolutionary Computation (CEC)*, 2021: IEEE, pp. 841-848.
[46] J. M. Rich and J. L. DeVitis, "Competition in Education," 1992.
[47] D. E. J. S. o. E. Merten, "The meaning of meanness: Popularity, competition, and conflict among junior high school girls," pp. 175-191, 1997.
[48] X. J. J. o. E. P. Wu, "The power of positional competition and market mechanism: A case study of recent parental choice development in China," vol. 23, no. 6, pp. 595-614, 2008.
[49] X. Zhou, A. Mau, and L. J. R. P. i. E. Jordan, "Gaming the no-choice system? School choice and persistent educational inequality in China," vol. 37, no. 1, pp. 134-152, 2022.
[50] S. J. A. i. e. s. Mirjalili, "The ant lion optimizer," vol. 83, pp. 80-98, 2015.
[51] L. Abualigah, M. Shehab, M. Alshinwan, H. J. N. C. Alabool, and Applications, "Salp swarm algorithm: a comprehensive survey," vol. 32, pp. 11195-11215, 2020.
[52] L. Abualigah, A. Diabat, S. Mirjalili, M. Abd Elaziz, A. H. J. C. m. i. a. m. Gandomi, and engineering, "The arithmetic optimization algorithm," vol. 376, p. 113609, 2021.
[53] S. J. K.-b. s. Mirjalili, "SCA: a sine cosine algorithm for solving optimization problems," vol. 96, pp. 120-133, 2016.
[54] H. Jia, X. Peng, and C. J. E. S. w. A. Lang, "Remora optimization algorithm," vol. 185, p. 115665, 2021.
[55] X. Liu, H. Huang, and J. Xiang, "A personalized diagnosis method to detect faults in gears using numerical simulation and extreme learning machine," *Knowledge-Based Systems,* vol. 195, p. 105653, 2020/05/11/ 2020, doi: https://doi.org/10.1016/j.knosys.2020.105653.
[56] J. Li and J. Lin, "A probability distribution detection based hybrid ensemble QoS prediction approach," *Information Sciences,* vol. 519, pp. 289-305, 2020/05/01/ 2020, doi: https://doi.org/10.1016/j.ins.2020.01.046.
[57] P. Zhou, L. Du, X. Liu, Y. D. Shen, M. Fan, and X. Li, "Self-Paced Clustering Ensemble," *IEEE Transactions on Neural Networks and Learning Systems,* vol. 32, no. 4, pp. 1497-1511, 2021, doi: 10.1109/TNNLS.2020.2984814.
[58] M. Fan, X. Zhang, J. Hu, N. Gu, and D. Tao, "Adaptive Data Structure Regularized Multiclass Discriminative Feature Selection," *IEEE transactions on neural networks and learning systems,* vol. PP, 04/21 2021, doi: 10.1109/TNNLS.2021.3071603.
[59] A. Kumar *et al.*, "A test-suite of non-convex constrained optimization problems from the real-world and some baseline results," vol. 56, p. 100693, 2020.
[60] A. Tzanetos and M. J. E. A. o. A. I. Blondin, "A qualitative systematic review of metaheuristics applied to tension/compression spring design problem: Current situation, recommendations, and research direction," vol. 118, p. 105521, 2023.
[61] L. Chen, F. Y. Xu, K. Z. Jin, and Z. Z. Tang, "Energy-saving access point configurations in WLANs: a swarm

intelligent approach," *JOURNAL OF SUPERCOMPUTING,* vol. 79, no. 17, pp. 19332-19364, 2023 NOV 2023, doi: 10.1007/s11227-023-05402-0.

[62] M. R. Shi, C. Chen, L. Liu, F. J. Kuang, D. Zhao, and X. W. Chen, "A grade-based search adaptive random slime mould optimizer for lupus nephritis image segmentation," *COMPUTERS IN BIOLOGY AND MEDICINE,* vol. 160, 2023 JUN 2023, Art no. 106950, doi: 10.1016/j.compbiomed.2023.106950.

[63] A. Lakhan, M. A. Mohammed, S. Kadry, S. A. AlQahtani, M. S. Maashi, and K. H. Abdulkareem, "Federated Learning-Aware Multi-Objective Modeling and blockchain-enable system for IIoT applications," *Computers and Electrical Engineering,* vol. 100, p. 107839, 2022/05/01/ 2022, doi: https://doi.org/10.1016/j.compeleceng.2022.107839.

[64] M. Hussain, L.-F. Wei, F. Abbas, A. Rehman, M. Ali, and A. Lakhan, "A multi-objective quantum-inspired genetic algorithm for workflow healthcare application scheduling with hard and soft deadline constraints in hybrid clouds," *Applied Soft Computing,* vol. 128, p. 109440, 2022/10/01/ 2022, doi: https://doi.org/10.1016/j.asoc.2022.109440.